

## GENERALIZED $p$ -ZBĀGANU CONSTANT IN BANACH SPACES

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**Abstract.** In this paper, we introduce generalized  $p$ -Zbăganu constant and discuss its bounds. Also, we obtain its relation with the generalized von Neumann-Jordan constant, the generalized Ptolemy constant, and the modulus of convexity. Furthermore, using generalized  $p$ -Zbăganu constant, we characterize the uniformly non-square and the weak normal structure and obtain that a Banach space has the fixed point property.

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