

SOME ASPECTS OF NEW SKEW GEOMETRIC CONSTANTS IN BANACH SPACES

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Abstract. In this paper, two new skew geometric constants are introduced. These constants are used to characterize Hilbert spaces. Some basic properties of these constants in Banach spaces are derived, and the values of the constants in specific spaces are calculated. On this basis, the relationships between the new geometric constants and other famous constants are studied. Finally, based on these identities, the relationship between the new geometric constants and the geometric properties in Banach spaces is discussed, such as uniform non-square and normal structure.

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