

NEW PERSPECTIVES ON BILATERAL INEQUALITIES FOR FUSION FRAMES IN HILBERT SPACES

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Abstract. We provide a new approach to the proofs of some existing bilateral inequalities for fusion frames in Hilbert spaces from the perspective of function theory, which greatly simplifies the proving process and shows that the parameter involving in four of the results mentioned can take values from larger ranges. We also present an improvement to two results on this topic. At the end of the paper we establish several new bilateral inequalities for fusion frames in Hilbert spaces, following the approaches of which corresponding bilateral inequalities for some other generalized frames with new types of structures can be naturally obtained.

Mathematics subject classification (2020): 42C15, 42C40, 46C50, 47B40.

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