

BOUNDS AND MONOTONICITY OF THE NIELSEN BETA FUNCTION

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Abstract. In this paper, we establish bounds and prove the complete monotonicity of functions involving the Nielsen beta function. We derive a novel integral representation for the Dirichlet eta function. As an application, we construct several bounded Bernstein functions associated with the Nielsen beta function.

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