

## WEIGHTED MAXIMAL POLYA–KNOPP INEQUALITIES

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**Abstract.** We characterize the pairs of weights  $(u, v)$  for the maximal operator  $G_0$ , defined for nonnegative functions on  $(0, \infty)$  by

$$G_0 f(x) = \sup_{b>x} \exp\left(\frac{1}{b} \int_0^b \log f\right),$$

to be bounded from  $L^p(v)$  to  $L^q(u)$ ,  $p \leq q$ , or from  $L^p(v)$  to  $L^{q,\infty}(u)$ .

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