

THE HOFFMAN-WIELANDT INEQUALITY FOR QUATERNION MATRICES AND QUATERNION MATRIX POLYNOMIALS

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Abstract. The purpose of this paper is to derive the Hoffman-Wielandt inequality and its generalization for quaternion matrices. Diagonalizability of the block companion matrix of certain quadratic (linear) quaternion matrix polynomials is brought out. As a consequence, we prove that if $Q(\lambda)$ is another quadratic (linear) quaternion matrix polynomial, then under certain conditions on the coefficients, a generalization of the Hoffman-Wielandt inequality for their corresponding block companion matrices holds. We also prove that if $P(\lambda)$ is a quaternion matrix polynomial with unitary coefficients, then any right eigenvalue λ_0 of $P(\lambda)$ lies in the annular region $\frac{1}{2} < |\lambda_0| < 2$.

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