

SHARP IMPROVEMENTS TO THE YOUNG INEQUALITY WITH THE KANTOROVICH CONSTANT AND APPLICATIONS

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Abstract. The aim of the present paper is to show the best possible bound of quadratic fractional type for one-term refinements and reverses of the Young inequality involving the Kantorovich constant. The key tool for obtaining the results is a new L' Hôpital-type higher monotone rule. Some applications to operator inequalities and to the theory of matrices are also discussed.

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