

## A CLASS OF DISCRETE HILBERT-TYPE INEQUALITIES IN THE WHOLE PLANE WITH A NON-MONOTONIC KERNEL

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**Abstract.** In this work, we first construct a non-monotonic discrete kernel function in the whole plane, where the parameters in the newly constructed kernel function are restricted to two special subset of the real line. Utilizing some classic techniques from real analysis and converting the weight functions in the whole plane to the first quadrant, we established the estimation formula for the weight functions, and then a class of new Hilbert-type inequality and its equivalent forms are established. In addition, it is proved that the constant factor of the newly obtained inequality is optimal. Moreover, assigning some special values to the parameters in the kernel function, some special inequalities are proved at the end of the paper.

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### REFERENCES

- [1] V. ADIYASUREN, T. BATBOLD AND M. KRNIĆ, *Half-discrete Hilbert-type inequalities with mean operators, the best constants, and applications*, Appl. Math. Comput., **231**, 3 (2014), 148–159.
- [2] T. BATBOLD, M. KRNIĆ, J. PEČARIĆ AND P. VUKOVIĆ, *Further development of Hilbert-type inequalities*, Element Press, Zagreb, 2017.
- [3] M. Z. GAO AND B. C. YANG, *On the extended Hilbert's inequality*, Proc. Amer. Math. Soc., **126**, 3 (1998), 751–759.
- [4] G. H. HARDY, J. E. LITTLEWOOD, G. PÓLYA, *Inequalities*, Cambridge Univ. Press, London, 1952.
- [5] G. H. HARDY, *Note on a theorem of Hilbert concerning series of positive terms*, Proc. Lond. Math. Soc., **23**, 2 (1925), 45–46.
- [6] L. C. HSU AND Y. J. WANG, *A refinement of Hilbert double series theorem*, J. Math. Res. Exp., **11**, 1 (1991), 143–144.
- [7] Y. HONG AND B. HE, *The theory and applications of Hilbert-type inequalities*, Science Press, Beijing, 2023.
- [8] M. KRNIĆ AND J. PEČARIĆ, *Extension of Hilbert's inequality*, J. Math. Anal. Appl., **324**, (2006), 150–160.
- [9] M. KRNIĆ AND J. PEČARIĆ, *Hilbert's inequalities and their reverses*, Publ. Math. Debrecen, **67**, 3–4 (2005), 315–331.
- [10] M. KRNIĆ, J. PEČARIĆ, I. PERIĆ AND P. VUKOVIĆ, *Recent advances in Hilbert-type inequalities*, Element Press, Zagreb, 2012.
- [11] M. KRNIĆ, J. PEČARIĆ AND P. VUKOVIĆ, *A unified treatment of half-discrete Hilbert-type inequalities with a homogeneous kernel*, Mediter. J. Math., **10**, (2013), 1697–1716.
- [12] J. C. KUANG AND L. DEBNATH, *On new generalizations of Hilbert's inequality and their applications*, J. Math. Anal. Appl., **245**, (2000), 248–265.
- [13] J. C. KUANG, *On new extension of Hilbert's integral inequality*, J. Math. Anal. Appl., **235**, (1999), 608–614.
- [14] B. G. PACHPATTE, *On some new inequalities similar to Hilbert's inequality*, J. Math. Anal. Appl., **226**, (1999), 166–179.
- [15] B. G. PACHPATTE, *Mathematical Inequalities*, Elsevier B. V., Netherland, 2005.

- [16] M. T. RASSIAS AND B. C. YANG, *A Hilbert-type integral inequality in the whole plane related to the hypergeometric function and the beta function*, J. Math. Anal. Appl., **428**, 4 (2015), 1286–1308.
- [17] Z. X. WANG AND D. R. GUO, *Introduction to Special Functions*, Higher Education Press, Beijing, 2012.
- [18] D. M. XIN, B. C. YANG AND A. Z. WANG, *Equivalent property of a Hilbert-type integral inequality related to the beta function in the whole plane*, J. Funct. Spaces, (2018), ID2691816, 8 pages.
- [19] D. M. XIN, B. C. YANG AND L. P. HE, *A new Hilbert-type inequality in the whole plane*, J. Math. Inequal., **17**, 4 (2023), 1521–1538.
- [20] B. C. YANG, *On an extension of Hardy-Hilbert's inequality*, Chinese Ann. Math. Ser. A, **23**, 2 (2002), 247–254.
- [21] B. C. YANG AND L. DEBNATH, *On a new generalization of Hardy-Hilbert's inequality and its application*, J. Math. Anal. Appl., **23**, 2 (1999), 484–497.
- [22] B. C. YANG, *The norm of operator and Hilbert-type inequalities*, Science Press, Beijing, 2009.
- [23] B. C. YANG AND M. T. RASSIAS, *On the way of weight coefficients and research for the Hilbert-type inequalities*, Math. Inequal. Appl., **6**, 2 (2003), 625–658.
- [24] B. C. YANG, *An extension of the Hilbert-type inequality and its reverse*, J. Math. Inequal., **2**, 1 (2008), 139–149.
- [25] B. C. YANG, *On a more accurate Hardy-Hilbert's type inequality and its applications*, Acta Math., **49**, 2 (2006), 363–368.
- [26] M. H. YOU, *On a new discrete Hilbert-type inequality and application*, Math. Inequal. Appl., **18**, 4 (2015), 1575–1578.
- [27] M. H. YOU, *On an Extension of the Discrete Hilbert Inequality and Applications*, J. Wuhan Univ. Natur. Sci. Ed., **67**, 2 (2021), 179–184.
- [28] M. H. YOU, *A unified extension of some classical Hilbert-type inequalities and applications*, Rocky Mountain J. Math., **51**, 5 (2021), 1865–1877.
- [29] M. H. YOU AND X. SUN, *On a Hilbert-type inequality with the kernel involving extended Hardy operator*, J. Math. Inequal., **15**, 3 (2021), 1239–1253.
- [30] M. H. YOU, *More accurate and strengthened forms of half-discrete Hilbert inequality*, J. Math. Anal. Appl., **512**, 2 (2022), [10.1016/j.jmaa.2022.126141](https://doi.org/10.1016/j.jmaa.2022.126141).
- [31] M. H. YOU, F. DONG AND Z. H. HE, *A Hilbert-type inequality in the whole plane with the constant factor related to some special constants*, J. Math. Inequal., **16**, 1 (2022), 35–50.
- [32] M. H. YOU, *On a class of Hilbert-type inequalities in the whole plane involving some classical kernel functions*, Proc. Edinb. Math. Soc., **65**, 3 (2022), 833–846.
- [33] Z. ZENG AND Z. T. XIE, *On a new Hilbert-type intergral inequality with the intergral in whole plane*, J. Inequal. Appl., 2010, 256796 (2010), <https://doi.org/10.1155/2010/256796>.