

THE CLOSURE OF THE ANALYTIC TYPE TENT SPACES IN THE WEIGHTED-TYPE SPACE AND GENERALIZED WEIGHTED COMPOSITION OPERATORS

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Abstract. In this paper, the closure of analytic type tent spaces within the weighted-type space is characterized. The boundedness and compactness of generalized weighted composition operators on the closure of analytic type tent spaces in the weighted-type space are studied. Additionally, the boundedness, compactness, and essential norm of generalized weighted composition operators from the weighted-type space to analytic type tent spaces are also investigated.

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