

ON A SUFFICIENT CONDITION FOR STARLIKENESS FOR MULTIVALENT FUNCTIONS

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Abstract. We consider univalent functions, analytic in the unit disc $|z| < 1$ in the complex plane \mathbb{C} which map $|z| < 1$ onto a domain with some property. In M. Nunokawa, J. Sokół, E. Trybucka, On some sufficient conditions for a function to be p -valent starlike, Symmetry, 11 (11) (2019), Art. 1417, we have established a sufficient condition for a function analytic in $|z| < 1$ to be p -valent starlike in $|z| < 1$. In this paper we shall determine the new sufficient conditions for the starlikeness of order α for p -valent functions and some symmetric results.

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