

ESTIMATES FOR MAXIMAL FOURIER MULTIPLIER OPERATORS ON \mathbb{R}^2 VIA SQUARE FUNCTIONS

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Abstract. We consider certain Littlewood-Paley square functions on \mathbb{R}^2 and prove sharp estimates for them, from which we can deduce L^p boundedness of maximal functions defined by Fourier multipliers of Bochner-Riesz type on \mathbb{R}^2 . This is a generalization of a result due to A. Carbery 1983.

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REFERENCES

- [1] A. CARBERY, *The boundedness of the maximal Bochner-Riesz operators on $L^4(\mathbb{R}^2)$* , Duke math. J., **50** (1983), 409–416.
- [2] A. CARBERY, *A weighted inequality for the maximal Bochner-Riesz operator on \mathbb{R}^2* , Trans. Amer. Math. Soc., **287** (1985), 673–680.
- [3] L. CARLESON AND P. SJÖLIN, *Oscillatory integrals and a multiplier problem for the disc*, Studia Math., **44** (1972), 287–299.
- [4] L. CLADEK, *On the square function associated with generalized Bochner-Riesz means*, 2015, arXiv:1510.05199v2 [math.CA].
- [5] A. CÓRDOBA, *A note on Bochner-Riesz operators*, Duke Math. J., **46** (1979), 505–511.
- [6] A. CÓRDOBA AND B. LÓPEZ-MELERO, *Spherical summation: a problem of E. M. Stein*, Ann. Inst. Fourier, Grenoble, **31** (1981), 147–152.
- [7] A. CÓRDOBA, *Vector valued inequalities for multipliers*, Conference on harmonic analysis in honor of Antoni Zygmund, Vol. I, II (Chicago, Ill., 1981), pp. 295–305, Wadsworth Math. Series, 1983.
- [8] C. FEFFERMAN, *The multiplier problem for the ball*, Annals of Math., **94** (1971), 330–336.
- [9] C. FEFFERMAN, *A note on spherical summation multipliers*, Israel J. Math., **15** (1973), 44–52.
- [10] L. HÖRMANDER, *Oscillatory integrals and multipliers on FL^p* , Ark. Mat., **11** (1973), 1–11.
- [11] N. H. KATZ, *Maximal operators over arbitrary sets of directions*, Duke Math. J., **97** (1999), 67–79.
- [12] S. SATO, *Some weighted estimates for Littlewood-Paley functions and radial multipliers*, J. Math. Anal. Appl., **278** (2003), 308–323.
- [13] S. SATO, *Weighted inequalities for Fourier multiplier operators of Bochner-Riesz type on \mathbb{R}^2* , Acta Math. Hungar., **171** (2) (2023), 301–324.
- [14] A. SEEGER AND S. ZIESLER, *Riesz means associated with convex domains in the plane*, Math. Z., **236** (2001), 643–676.
- [15] P. SJÖLIN, *Fourier multipliers and estimates of the Fourier transform of measures carried by smooth curves in \mathbb{R}^2* , Studia Math., **51** (1974), 169–182.
- [16] C. D. SOGGE, *Fourier Integrals in Classical Analysis*, Cambridge University Press, 1993.
- [17] E. M. STEIN AND G. WEISS, *Introduction to Fourier Analysis on Euclidean Spaces*, Princeton Univ. Press, 1971.
- [18] JAN-OLOF STRÖMBERG, *Maximal functions associated to rectangles with uniformly distributed directions*, Annals of Math., **107** (1978), 399–402.