

INEQUALITIES BETWEEN IDENTRIC MEAN AND CONVEX COMBINATIONS OF OTHER MEANS

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Abstract. By using the asymptotic expansion method, Elezović conjectured inequalities between identric mean and convex combinations of other means. In this paper, we prove certain conjectures given by Elezović.

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