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ON THE OPERATOR EQUATION $AXB + CYD = Z$

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Abstract. Suppose two n-tuples of operators A_k and B_k on a Hilbert space are given, and Φ is the mapping from the set of n-tuples of operators on the Hilbert space into the set of all operators on the space defined by $\Phi(X_1, X_2, \ldots, X_n) = \sum_{k=1}^{n} A_k X_k B_k$. Conditions are given for Φ to be onto.

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REFERENCES

[1] RAJENDRA BHATIA AND PETER ROSENTHAL, *How and why to solve the operator equation* $AX - XB = Y$, Bull London Math. Soc. **29** (1997), 1–21. MR1416400 (97k:47016)
[2] M. BRESAR AND PETER SEMRL, *Elementary operators*, Proc. Royal Soc. Edinburgh Sect. A, **129** (1999), 1115-1135. MR1728537 (2001f:4701)
[3] CHANDLER DAVIS AND PETER ROSENTHAL, *Solving linear operator equations*, Canadian J. Math. **26** (1974), 1384–1389. MR0355649 (50 #8123)
[4] RONALD G. DOUGLAS, *On majorization, factorization, and range inclusion of operators on Hilbert space*, Proc. Amer. Math. Soc., **17** (1966), 413–415. MR0203464 (34 #3315)
[5] PETER FILLMORE, JOSEPH STAMPFLI AND JAMES WILLIAMS, *On the essential numerical range, the essential spectrum, and a problem of Halmos*, Acta Math. (Szeged) **33** (1972), 179–192. MR0322534 (48 #896)
[6] ERIC NORDGREN AND PETER ROSENTHAL, *Boundary values of Berezin symbols*, Operator Theory: Advances and Applications, **73** (1994), 362–368. MR1320554 (96b:46036)
[7] MARVIN ROSENBLUM, *On the operator equation* $BX - XA = Q$, Duke Math. J. **23** (1956), 263–270. MR0079235 (18, 54d)

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