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THE KALMAN–YAKUBOVICH–POPOV INEQUALITY FOR PASSIVE DISCRETE TIME–INVARIANT SYSTEMS

YURY ARLINSKIĬ

Abstract. We consider the Kalman - Yakubovich - Popov (KYP) inequality

$$\begin{pmatrix} X - A^*XA - C^*C & -A^*XB - C^*D \\ -B^*XA - D^*C & I - B^*XB - D^*D \end{pmatrix} \geqslant 0$$

for contractive operator matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix} : \begin{pmatrix} \mathfrak{H} \\ \mathfrak{M} \end{pmatrix} \to \begin{pmatrix} \mathfrak{H} \\ \mathfrak{N} \end{pmatrix}$, where $\mathfrak{H}$, $\mathfrak{M}$, and $\mathfrak{N}$ are separable Hilbert spaces. We restrict ourselves to the positive contractive solutions X. Using the parametrization of the blocks of contractive operator matrices, the Kreĭn shorted operator, and the Möbius representation of the Schur class operator-valued function we find several equivalent forms of the KYP inequality. The properties of solutions are established and it is proved that the minimal solution of the KYP inequality satisfies the corresponding algebraic Riccati equation and can be obtained by the iterative procedure with the special choice of the initial point. In terms of the Kreĭn shorted operators the necessary condition and some sufficient conditions for uniqueness of the solution are established.

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