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A NOTE ON FREE PRODUCTS

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Abstract. We answer two questions raised by I. M. Singer concerning free products. We prove that in a free product of separable unital C*-algebras, states on each algebra can be simultaneously extended to a pure state on the free product. We also show that the second dual of the free product of unital C*-algebras is the von Neumann algebra free product of their second duals. We give a proof that the extreme points of the set of tracial states of a C*-algebra is the set of factor tracial states.

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