

CHARACTERIZING JORDAN AUTOMORPHISMS OF MATRIX ALGEBRAS THROUGH PRESERVING PROPERTIES

PETER ŠEMRL

Abstract. Let M_n be the algebra of all $n \times n$ complex matrices, $n \geq 3$. We prove that a map $\phi : M_n \rightarrow M_n$ is a Jordan automorphism if and only if ϕ is a continuous spectrum and commutativity preserving map (no linearity is assumed). Examples are given showing that this characterization is optimal

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