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FREDHOLMNESS AND INDEX OF OPERATORS IN THE WIENER ALGEBRA ARE INDEPENDENT OF THE UNDERLYING SPACE

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Abstract. The purpose of this paper is to demonstrate the so-called Fredholm-inverse closedness of the Wiener algebra $\mathscr{W}$ and to deduce independence of the Fredholm property and index of the underlying space. More precisely, we look at operators $A \in \mathscr{W}$ as acting on a family of vector valued ℓ^p spaces and show that the Fredholm regularizer of A for one of these spaces can always be chosen in $\mathscr{W}$ as well and therefore regularizes A (modulo compact operators) on all of the ℓ^p spaces under consideration. We conclude that both Fredholmness and the index of A do not depend on the ℓ^p space that A is considered as acting on.

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