

## A NEW CHARACTERIZATION OF THE CLOSURE OF THE ( $\mathcal{U} + \mathcal{K}$ )-ORBIT OF CERTAIN ESSENTIALLY NORMAL OPERATORS

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*Abstract.* The  $(\mathcal{U} + \mathcal{K})$ -orbit of a Hilbert space operator  $T$  is defined as  $(\mathcal{U} + \mathcal{K})(T) = \{ R^{-1}TR : R \text{ invertible of the form unitary plus compact} \}$ . In this paper, we show that certain essentially normal operator with the same spectral picture as an essentially normal injective unilateral weighted operator generates the same closure of  $(\mathcal{U} + \mathcal{K})$ -orbit.

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