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A NOTE ON APPROXIMATE LIFTINGS

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Abstract. In this paper, we prove approximate lifting results in the C^*-algebra and von Neumann algebra settings. In the C^*-algebra setting, we show that two (weakly) semiprojective unital C*-algebras, each generated by n projections, can be glued together with partial isometries to define a larger (weakly) semiprojective algebra. In the von Neumann algebra setting, we prove lifting theorems for trace-preserving *-homomorphisms from abelian von Neumann algebras or hyperfinite von Neumann algebras into ultraproducts. We also extend a classical result of S. Sakai [16] by showing that a tracial ultraproduct of C*-algebras is a von Neumann algebra, which yields a generalization of Lin's theorem [12] on almost commuting selfadjoint operators with respect to $\|\cdot\|_p$ on any unital C*-algebra with trace.

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