

## NEAREST SOUTHEAST SUBMATRIX THAT MAKES MULTIPLE AN EIGENVALUE OF THE NORMAL NORTHWEST SUBMATRIX

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**Abstract.** Let  $A, B, C, D$  be four complex matrices, where  $D \in \mathbb{C}^{m \times m}$  and  $A \in \mathbb{C}^{n \times n}$  is a normal matrix. Let  $z_0$  be an fixed eigenvalue of  $A$ . We find the distance (with respect to the 2-norm) from  $D$  to the set of matrices  $X \in \mathbb{C}^{m \times m}$  such that  $z_0$  is a multiple eigenvalue of the matrix

$$\begin{pmatrix} A & B \\ C & X \end{pmatrix}.$$

We also give an expression for one of the closest matrices.

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