

ORTHOGONALLY ADDITIVE HOLOMORPHIC FUNCTIONS ON C^* -ALGEBRAS

ANTONIO M. PERALTA AND DANIELE PUGLISI

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Abstract. Let A be a C^* -algebra. We prove that a holomorphic function of bounded type $f : A \rightarrow \mathbb{C}$ is orthogonally additive on A_{sa} if, and only if, it is additive on elements having zero-product if, and only if, there exist a positive functional φ in A^* , a sequence (ψ_n) in $L_1(A^{**}, \varphi)$ and a power series holomorphic function h in $\mathcal{H}_b(A, A^*)$ such that

$$h(a) = \sum_{k=1}^{\infty} \psi_k \cdot a^k \text{ and } f(a) = \langle 1_{A^{**}}, h(a) \rangle = \int h(a) d\varphi,$$

for every a in A , where $1_{A^{**}}$ denotes the unit element in A^{**} and $L_1(A^{**}, \varphi)$ is a non-commutative L_1 -space.

1. Introduction

Let A be a C^* -algebra whose self-adjoint part is denoted by A_{sa} . Two elements a and b in A are said to be *orthogonal* (denoted by $a \perp b$) if $ab^* = b^*a = 0$. When $ab = 0 = ba$ we shall say that a and b have *zero-product*.

Let A be a C^* -algebra and let X be a complex Banach space. A mapping $f : A \rightarrow X$ is said to be *orthogonally additive* (respectively, *orthogonally additive on A_{sa}*) if for every a, b in A (respectively, a, b in A_{sa}) with $a \perp b$ we have $f(a+b) = f(a) + f(b)$. We shall say that f is *additive on elements having zero-product* if for every a, b in A with $ab = 0 = ba$ we have $f(a+b) = f(a) + f(b)$.

Let X and Y be Banach spaces. A (continuous) m -homogeneous polynomial P from X to Y is a mapping $P : X \rightarrow Y$ for which there is a (continuous) multilinear symmetric operator $A : X \times \dots \times X \rightarrow Y$ such that $P(x) = A(x, \dots, x)$, for every $x \in X$. All the polynomials considered in this paper are assumed to be continuous.

Orthogonally additive n -homogeneous polynomials over $C(K)$ -spaces and Banach lattices have been independently studied by Y. Benyamini, S. Lassalle and J.G. Llavona (cf. [1]) and D. Pérez and I. Villanueva (cf. [9]); a short proof was published by

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D. Carando, S. Lassalle and I. Zalduendo [3]. Orthogonally additive n -homogeneous polynomials over general C^* -algebras were described by C. Palazuelos, I. Villanueva and the first author of this note in [8] (see also [2, §3]). These results extend the characterization given by K. Sundaresan for L^p -spaces [12]. In the setting of C^* -algebras we have:

THEOREM 1. [8, Theorem 2.8 and Corollary 3.1] *Let A be a C^* -algebra, X a Banach space, $n \in \mathbb{N}$, and P an n -homogeneous polynomial from A to X . The following are equivalent.*

(a) *There exists a bounded linear operator $T : A \rightarrow X$ satisfying*

$$P(x) = T(x^n),$$

for every $x \in A$, and $\|P\| \leq \|T\| \leq 2\|P\|$.

(b) *P is additive on elements having zero-products.*

(c) *P is orthogonally additive on A_{sa} .*

A mapping f from a Banach space X to a Banach space Y is said to be *holomorphic* if for each $x \in X$ there exists a sequence of polynomials $P_k(x) : X \rightarrow Y$, and a neighbourhood V_x of x such that the series

$$\sum_{k=0}^{\infty} P_k(x)(y-x)$$

converges uniformly to $f(y)$ for every $y \in V_x$. A holomorphic function $f : X \rightarrow Y$ is said to be of *bounded type* if it is bounded on all bounded subsets of X , in this case its Taylor series at zero, $f = \sum_{k=0}^{\infty} P_k$, has infinite radius of uniform convergence, i.e. $\limsup_{k \rightarrow \infty} \|P_k\|^{\frac{1}{k}} = 0$ (compare [5, §6.2]). We refer to [5] for the basic facts and definitions used in this paper.

Homogeneous polynomials on a C^* -algebra A are the simplest examples of holomorphic functions on A .

In a recent paper, D. Carando, S. Lassalle and I. Zalduendo considered orthogonally additive holomorphic functions of bounded type from $C(K)$ to \mathbb{C} (cf. [4]). These authors noticed that the characterizations obtained for orthogonally additive n -homogeneous polynomial can not be expected for orthogonally additive holomorphic functions of bounded type from $C(K)$ to \mathbb{C} . They actually show that there is no entire function $\Phi : \mathbb{C} \rightarrow \mathbb{C}$ such that the mapping $h : C(K) \rightarrow C(K)$, $h(f) = \Phi \circ f$ factors all degree-2 orthogonally additive scalar polynomials over $C(K)$. In the same paper, the authors quoted above gave an alternative characterization of all orthogonally additive scalar holomorphic functions of bounded type over $C(K)$ -spaces.

We recall that, given a Borel regular measure μ on a compact Hausdorff space K , a holomorphic function h in $\mathcal{H}_b(C(K), L_1(\mu))$ is a *power series function* if there exists a sequence $(g_k)_k \subseteq L_1(\mu)$ such that

$$h(x) = \sum_{k=0}^{\infty} g_k x^k, \quad (x \in C(K)).$$

The following theorem is due to D. Carando, S. Lassalle and I. Zalduendo.

THEOREM 2. [4, Theorem 3.3] *Let K be a compact Hausdorff space. A holomorphic function of bounded type $f : C(K) \rightarrow \mathbb{C}$ is orthogonally additive if and only if there exist a Borel regular measure μ on K and a power series function $h \in \mathcal{H}_b(C(K), L_1(\mu))$ such that*

$$f(x) = \int_K h(x) \, d\mu,$$

for every $x \in C(K)$.

The above theorem can be read as follows: A holomorphic function of bounded type $f : C(K) \rightarrow \mathbb{C}$ is orthogonally additive if and only if there exist a Borel regular measure μ on K and a power series function $h \in \mathcal{H}_b(C(K), L_1(\mu))$ such that

$$f(x) = \langle 1_{C(K)}, h(x) \rangle, \quad x \in C(K).$$

Recently, J.A. Jaramillo, A. Prieto and I. Zalduendo introduced and studied “locally” orthogonally additive holomorphic functions defined on an open subset of $C(K)$ (see [6]).

In this note we study orthogonally additive scalar holomorphic functions of bounded type on a general C^* -algebra. In our main result (see Theorem 5) we prove that a scalar holomorphic function of bounded type f from a C^* -algebra A is orthogonally additive on A_{sa} if and only if there exist a positive functional φ in A^* , a sequence (ψ_n) in $L_1(A^{**}, \varphi)$ and a power series holomorphic function h in $\mathcal{H}_b(A, A^*)$ such that

$$h(a) = \sum_{k=1}^{\infty} \psi_k \cdot a^k \text{ and } f(a) = \langle 1_{A^{**}}, h(a) \rangle = \int h(a) \, d\varphi,$$

for every a in A , where $1_{A^{**}}$ denotes the unit element in A^{**} and $L_1(A^{**}, \varphi)$ is one of the non-commutative L_1 -spaces studied, for example, by H. Kosaki in [7].

2. Holomorphic mappings on general C^* -algebras

In this section we shall study orthogonally additive complex-valued holomorphic functions of bounded type on a general C^* -algebra.

The following lemma was essentially obtained in [4].

LEMMA 3. *Let $f : A \rightarrow X$ be a holomorphic function of bounded type from a C^* -algebra to a complex Banach space and let $f = \sum_{k=0}^{\infty} P_k$ be its Taylor series at zero. Then, f is orthogonally additive (respectively, orthogonally additive on A_{sa} or additive on elements having zero-product) if, and only if, all the P_k 's satisfy the same property. In such a case, $P_0 = 0$.*

Proof. The proof of [4, Lemma 1.1] remains valid here. \square

Before dealing with the main theorem of this section, we shall recall some technical results on non-commutative $L_p(\varphi)$ spaces which are needed later. The construction presented here is inspired by those results established by H. Kosaki and G. Pisier in [7, §3] and [10, §3], respectively.

For each element x in a C^* -algebra, A , the (*Jordan*) *modulus* of x is defined by

$$|x| := \left(\frac{x^*x + xx^*}{2} \right)^{\frac{1}{2}}.$$

We shall denote by A_+ the set of all positive elements in A .

Let φ be a positive functional in A^* . We may equip A with a scalar product defined by

$$(x, y)_\varphi = \varphi \left(\frac{x^*y + yx^*}{2} \right), (x, y \in A).$$

Let $N = N_\varphi := \{x \in A : \varphi(|x|^2) = 0\}$. Then, N is a norm-closed subspace of A . Let A^0 denote the Banach space A/N equipped with the quotient norm. The space $(A/N, (\cdot, \cdot)_\varphi)$ is a prehilbert space. The completion of $(A/N, (\cdot, \cdot)_\varphi)$ is a Hilbert space which shall be denoted by $L_2(A, \varphi)$ or simply by $L_2(\varphi)$.

Let $J_\varphi : A^0 \hookrightarrow L_2(\varphi)$ denote the natural embedding. It is clear that J_φ is a continuous operator which has norm dense range and $\|J_\varphi\| \leq \|\varphi\|^{\frac{1}{2}}$. Considering the injection $\iota = J_\varphi^* \circ J_\varphi : A^0 \rightarrow (A^0)^* \subseteq A^*$, the space $L_1(A, \varphi) = L_1(\varphi)$ is defined as the norm closure of $\iota(A^0)$ in A^* .

It should be also mentioned here that, for each $x + N \in A^0$

$$\iota(x + N) = \frac{x \cdot \varphi + \varphi \cdot x}{2}, \tag{1}$$

where $x \cdot \varphi, \varphi \cdot x$ defined as elements in A^* are given by

$$x \cdot \varphi(y) = \varphi(yx) \quad \text{and} \quad \varphi \cdot x(y) = \varphi(xy),$$

(compare [10, page 124]).

When A is a von Neumann algebra and φ is a positive element in the predual of A , for each element $a + N$ in A^0 , we have $\iota(a + N) = \frac{a \cdot \varphi + \varphi \cdot a}{2} \in A_*$. Since A_* is a norm closed subspace of A^* , we have $L_1(A, \varphi) \subseteq A_*$.

$L_1(\varphi)$ is a closed subspace of A^* , therefore given g in $L_1(\varphi)$, we can compute $g(1_{A^{**}}) = \langle g, 1_{A^{**}} \rangle$, where $1_{A^{**}}$ denotes the unit element in A^{**} . According to the notation employed in the abelian case, we shall write

$$\int g \, d\varphi = \langle g, 1_{A^{**}} \rangle.$$

In order to be consistent with the terminology used in the commutative setting, we shall say that a mapping $h : A \rightarrow A^*$ is a *power series* function if $h(a) = \sum_{k=0}^\infty g_k \cdot a^k$ or

$h(a) = \sum_{k=0}^{\infty} a^k \cdot g_k$, for every $a \in A$, where (g_k) is a sequence in A^* . Clearly, every power series function h is a holomorphic function of bounded type.

The following generalisation of Radon-Nikodym theorem, due to S. Sakai, will be needed later.

THEOREM 4. [11, Proposition 1.24.4] *Let \mathcal{M} be a von Neumann algebra, and let φ, ψ be two normal positive linear functionals on \mathcal{M} such that $\psi \leq \varphi$. Then, there exists a positive element t_0 in \mathcal{M} , with $0 \leq t_0 \leq 1$, such that*

$$\psi(x) = \frac{t_0 \cdot \varphi + \varphi \cdot t_0}{2}(x) = \frac{1}{2} \varphi(t_0 x + x t_0), \text{ for every } x \in \mathcal{M}.$$

Now, we are ready to state the main result of this section.

THEOREM 5. *Let $f : A \rightarrow \mathbb{C}$ be a holomorphic function of bounded type defined on a C^* -algebra. Then the following are equivalent:*

- (a) *f is orthogonally additive on A_{sa} .*
- (b) *f is additive on elements having zero-product.*
- (c) *There exist a positive functional φ in A^* and a power series holomorphic function h in $\mathcal{H}_b(A, A^*)$ such that*

$$f(a) = \langle 1_{A^{**}}, h(a) \rangle = \int h(a) d\varphi,$$

*for every a in A , where $1_{A^{**}}$ denotes the unit element in A^{**} .*

Proof. The implications (c) \Rightarrow (b) \Rightarrow (a) are clear. We shall prove (a) \Rightarrow (c).

Let $f = \sum_{k=1}^{\infty} P_k$ be the Taylor series expression of f at zero. Since f is orthogonally additive on A_{sa} , by Lemma 3, each k -homogeneous polynomial P_k is orthogonally additive on A_{sa} . Thus, by Theorem 1 (c.f. [8]), for each natural k , there exists $\varphi_k \in A^*$ such that

$$P_k(a) = \varphi_k(a^k) \quad (a \in A),$$

with

$$\|P_k\| \leq \|\varphi_k\| \leq 2\|P_k\|, \quad \forall k \in \mathbb{N}.$$

Let us write each φ_k in the form

$$\varphi_k = (\varphi_{k,1} - \varphi_{k,3}) + i(\varphi_{k,2} - \varphi_{k,4})$$

where $\varphi_{k,j} \in (A^*)^+$, for $j = 1, \dots, 4, k \in \mathbb{N}$.

Since $\|\varphi_{k,j}\| \leq \|\varphi_k\|$ for all $j = 1, \dots, 4$ and $k \in \mathbb{N}$, and the series $\sum_{k=0}^{\infty} \|P_k\| \lambda^k$ has infinite radius of convergence, the expression

$$\varphi = \sum_{k=1}^{\infty} \sum_{j=1}^4 \varphi_{k,j}$$

defines a positive functional in A^* .

Since, trivially, $\varphi_{k,j} \leq \varphi$, for each $j = 1, \dots, 4$ and $k \in \mathbb{N}$, by Theorem 4 (c.f. [11, Theorem 1.24.4]) applied to the von Neumann algebra A^{**} , for each $j = 1, \dots, 4$ and $k \in \mathbb{N}$, there exist $0 \leq t_{k,j} \leq 1$ in A^{**} such that

$$\varphi_{k,j}(a) = \varphi \left(\frac{t_{k,j} a + a t_{k,j}}{2} \right), \quad (a \in A^{**}).$$

Let us consider the space $L_1(A^{**}, \varphi)$ and the natural embedding

$$A/N \hookrightarrow A^{**}/\overline{N}^{w*} \xrightarrow{\iota} L_1(A^{**}, \varphi),$$

where $N = \{x \in A : \varphi(|x|^2) = 0\}$ and $\overline{N}^{w*} = \{x \in A^{**} : \varphi(|x|^2) = 0\}$. When (1) is applied to φ , considered as a functional on A^{**} , gives

$$\iota(a + \overline{N}^{w*}) = \frac{a \cdot \varphi + \varphi \cdot a}{2} \quad (a \in A^{**}).$$

For each $(j, k) \in \{1, \dots, 4\} \times \mathbb{N}$, $\iota(t_{k,j} + \overline{N}^{w*})$ is positive in A^* , and we have

$$\begin{aligned} \|\iota(t_{k,j} + \overline{N}^{w*})\|_{L_1(\varphi)} &= \langle \iota(t_{k,j} + \overline{N}^{w*}), 1_{A^{**}} \rangle = \iota(t_{k,j} + \overline{N}^{w*})(1_{A^{**}}) \\ &= \varphi(t_{k,j}) = \|\varphi_{k,j}\|_{A^*}. \end{aligned}$$

Let us define $h : A \longrightarrow L_1(A^{**}, \varphi) \cdot A \subseteq A^*$ by

$$h(a) = \sum_{k=1}^{\infty} \iota(t_k + \overline{N}^{w*}) \cdot a^k,$$

where, for each natural k , $t_k = (t_{k,1} - t_{k,3}) + i(t_{k,2} - t_{k,4})$ and for each element $x \in A$ and a functional $\psi \in A^*$, $\psi \cdot x$ denotes the element in A^* defined as $\psi \cdot x(y) = \psi(xy)$. It should be noticed here that $L_1(A^{**}, \varphi) \cdot A$ need not be, in general, a subset of $L_1(A^{**}, \varphi)$; we can only guarantee that $L_1(A^{**}, \varphi) \cdot A \subseteq A^*$.

In order to see that h is well defined, let us estimate

$$\begin{aligned} \sum_{k=1}^{\infty} \|\iota(t_k + \overline{N}^{w*}) \cdot a^k\|_{A^*} &\leq \sum_{k=1}^{\infty} \sum_{j=1}^4 \|\iota(t_{k,j} + \overline{N}^{w*})\|_{L_1(\varphi)} \|a\|_A^k \\ &= \sum_{k=1}^{\infty} \sum_{j=1}^4 \|\varphi_{k,j}\|_{A^*} \|a\|^k \\ &\leq 4 \sum_{k=1}^{\infty} \|\varphi_k\|_{A^*} \|a\|^k \leq 8 \sum_{k=1}^{\infty} \|P_k\| \|a\|^k < \infty \end{aligned}$$

for every a in A , where in the last inequality we applied that $\lim_{k \rightarrow \infty} \|P_k\|^{\frac{1}{k}} = 0$.

This tells us that h is a well defined power series holomorphic function of bounded type. It follows from the construction that

$$\begin{aligned} f(a) &= \sum_{k=1}^{\infty} P_k(a) = \sum_{k=1}^{\infty} \varphi_k(a^k) = \sum_{k=1}^{\infty} [(\varphi_{k,1} - \varphi_{k,3}) + i(\varphi_{k,2} - \varphi_{k,4})] (a^k) \\ &= \sum_{k=1}^{\infty} \sum_{j=1}^4 i^{(j-1)} \varphi \left(\frac{t_{k,j} a^k + a^k t_{k,j}}{2} \right) = \sum_{k=1}^{\infty} \sum_{j=1}^4 i^{(j-1)} \frac{t_{k,j} \cdot \varphi + \varphi \cdot t_{k,j}}{2} (a^k) \\ &= \sum_{k=1}^{\infty} \sum_{j=1}^4 i^{(j-1)} \iota(t_{k,j} + \overline{N}^{w*}) (a^k) = \sum_{k=1}^{\infty} \iota(t_k + \overline{N}^{w*}) (a^k) \\ &= \langle 1_{A^{**}}, \sum_{k=1}^{\infty} \iota(t_k + \overline{N}^{w*}) \cdot a^k \rangle = \langle 1_{A^{**}}, h(a) \rangle = \int h(a) d\varphi. \quad \square \end{aligned}$$

Let A be a C^* -algebra. We have proved that every scalar holomorphic function of bounded type on A which is orthogonally additive on A_{sa} factors through the norm-closure of $L_1(A^{**}, \varphi) \cdot A$ in A^* , where φ is a suitable positive functional in A^* . Under some additional hypothesis, we shall prove that we can get a factorization through a non-commutative L_1 space.

Let \mathcal{M} be a semi-finite von Neumann algebra with a faithful semi-finite normal trace τ (cf. [13, Theorem 2.15]). Let

$$m_\tau = \{xy : \tau(|x|^2), \tau(|y|^2) < \infty\},$$

then m_τ is a two-sided ideal of \mathcal{M} , called the *definition ideal* of the trace τ . The assignment $x \mapsto \|x\|_1 := \tau(|x|)$ defines a norm on m_τ . Actually, the space $(m_\tau, \|x\|_1)$ can be identified with a subspace of \mathcal{M}_* via the following norm-one bilinear form

$$\begin{aligned} \mathcal{M} \times m_\tau &\rightarrow \mathbb{C} \\ (a, x) &\mapsto \tau(ax). \end{aligned} \tag{2}$$

For each $x \in m_\tau$, the symbol ω_x will denote the functional defined by $\omega_x(y) := \tau(xy)$. $L_1(\mathcal{M}, \tau)$ is defined as the completion of $(m_\tau, \|\cdot\|_1)$. It is also known that, for each $x \in m_\tau$, $\|x\|_1 = \sup\{|\tau(yx)| : y \in \mathcal{M}, \|y\| \leq 1\}$. The bilinear form defined in (2) extends to a norm-one bilinear form on $\mathcal{M} \times L_1(\mathcal{M}, \tau)$, and $L_1(\mathcal{M}, \tau)$ is isometrically isomorphic to the predual \mathcal{M}_* (cf. [13, Pages 319-321]). In the literature, the von Neumann algebra \mathcal{M} is usually denoted by $L_\infty(\mathcal{M}, \tau)$.

This notation is coherent with the one used before. When τ is a normal faithful finite trace on \mathcal{M} then there exists a central (i.e. $\varphi(xy) = \varphi(yx)$ for every $x, y \in M$), positive and faithful functional φ in \mathcal{M}_* such that $\tau = \varphi|_{M^+}$. In this case, the spaces $L_1(\mathcal{M}, \tau)$ and $L_1(\mathcal{M}, \varphi)$ coincide.

THEOREM 6. *Let A be a C^* -algebra such that A^{**} is a semi-finite von Neumann algebra and let $f : A \rightarrow \mathbb{C}$ be a holomorphic function of bounded type. Suppose that τ is a faithful semi-finite normal trace on A^{**} . Then f is orthogonally additive on A_{sa}*

if, and only if, there exists a power series holomorphic function $h : A \rightarrow L_1(A^{**}, \tau)$ such that

$$f(a) = \langle h(a), 1_{A^{**}} \rangle = \int h(a) \, d\tau$$

for every a in A , where $1_{A^{**}}$ denotes the unit element in A^{**} .

Proof. Let $f = \sum_{k=1}^{\infty} P_k$ be the Taylor series of f at zero. Since f is orthogonally additive on A_{sa} , by Lemma 3, each k -homogeneous polynomial P_k is orthogonally additive on A_{sa} . Thus, by Theorem 1 (c.f. [8]), for each natural k , there exists $\varphi_k \in A^*$ such that

$$P_k(a) = \varphi_k(a^k) \quad (a \in A),$$

with

$$\|P_k\| \leq \|\varphi_k\| \leq 2\|P_k\|, \quad \forall k \in \mathbb{N}.$$

We have already mentioned that $(A^{**})_* = A^* = L_1(A^{**}, \tau)$. Since for each $k \in \mathbb{N}$, $\varphi_k \in A^*$, by construction, $\varphi_k = g_k \in L_1(A^{**}, \tau)$, with $\|g_k\|_1 = \|\varphi_k\|_{A^*}$.

Given $a \in A$ and $g \in L_1(A^{**}, \tau)$, there exists a sequence $(y_n)_n$ in \mathfrak{m}_τ such that $\|\omega_{y_n} - g\|_1 \rightarrow 0$, then the sequence $(\omega_{y_n a})_n$ is Cauchy in $L_1(A^{**}, \tau)$. The limit of $(\omega_{y_n a})_n$ is denoted by $g \cdot a$. Further, it is not hard to see that $g \cdot a(x) = g(ax)$, for all $x \in A$. In particular $\|g \cdot a\|_1 \leq \|g\|_1 \|a\|_A$.

Let us define

$$h : A \longrightarrow L_1(A^{**}, \tau)$$

the mapping given by

$$h(a) = \sum_{k=1}^{\infty} g_k \cdot a^k.$$

Since

$$\left\| \sum_{k=1}^{\infty} g_k \cdot a^k \right\|_1 \leq \sum_{k=1}^{\infty} \|g_k\|_1 \|a\|^k = \sum_{k=1}^{\infty} \|\varphi_k\|_{A^*} \|a\|^k < \infty,$$

we deduce that h is well defined and $f(a) = \langle h(a), 1_{A^{**}} \rangle = \int h(a) \, d\tau$, for all $a \in A$, where $1_{A^{**}}$ denotes the unit element in A^{**} . \square

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Antonio M. Peralta
 Departamento de Análisis Matemático
 Facultad de Ciencias
 Universidad de Granada
 18071 Granada, Spain
 e-mail: aperalta@ugr.es

and

Department of Mathematics and Computer Sciences
 University of Catania
 Catania, 95125, Italy
 e-mail: dpuglisi@dmi.unict.it

Daniele Puglisi
 Departamento de Análisis Matemático
 Facultad de Ciencias
 Universidad de Granada
 18071 Granada
 Spain
 e-mail: puglisi@math.kent.edu