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SPECTRAL PROPERTIES BETWEEN OPERATOR MATRICES AND HELTON CLASS

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Abstract. In this paper, we study properties of Helton class of an operator matrix. In particular, we show that some upper operator matrix belongs to Helton class of an operator matrix have the property $(\beta)_\varepsilon$. As an application, we get that such operators have nontrivial invariant subspaces. Finally, we prove that Helton class preserves the generalized Weyl's theorem under some conditions.

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