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THE BIRKHOFF ORTHOGONALITY IN PRE–HILBERT C^*–MODULES

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Abstract. In this work we characterize the Birkhoff orthogonality for elements and finite dimensional subspaces of a pre-Hilbert C^*-module in terms of a convex hull of continuous linear functionals. The aim of the paper is to present results concerning the B-orthogonality and its applications. We also present the results concerning smoothness. Moreover, we give a new proof of the Bhatia–Šemrl theorem.

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REFERENCES

[1] LJ. ARAMBAŠIĆ AND R. RAJIĆ, *The Birkhoff–James orthogonality in Hilbert C^*-modules*, Linear Algebra Appl. **437** (2012), no. 7, 1913–1929.
[2] R. BHATIA, P. ŠEMRL, *Orthogonality of matrices and some distance problems*, Linear Algebra Appl. **287** (1999), no. 1–3, 77–85.
[3] T. BHATTACHARYYA, P. GROVER, *Characterization of Birkhoff–James orthogonality*, J. Math. Anal. Appl. **407** (2013), 350–358.
[4] G. BIRKHOFF, *Orthogonality in linear metric spaces*, Duke Math. J. **1** (1935), 169–172.
[5] S. S. DRAGOMIR, *Semi-Inner Products and Applications*, Nova Science Publishers, Inc., Hauppauge, NY, 2004.
[6] R. C. JAMES, *Orthogonality and linear functionals in normed linear spaces*, Trans. Amer. Math. Soc., **61** (1947), 265–292.
[7] I. SINGER, *Best Approximation in Normed Linear Spaces by Elements of Linear Subspaces*, Springer-Verlag, Berlin, Heidelberg, New York, 1970.

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