

# ASYMMETRIC TRUNCATED TOEPLITZ OPERATORS ON FINITE-DIMENSIONAL SPACES

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**Abstract.** In this paper we consider asymmetric truncated Toeplitz operators acting between two finite-dimensional model spaces. We compute the dimension of the space of all such operators. We also describe the matrix representations of asymmetric truncated Toeplitz operators acting between two finite-dimensional model spaces. Our results are generalizations of the results known for truncated Toeplitz operators.

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