

## MAPS PRESERVING THE PERIPHERAL LOCAL SPECTRUM OF SOME PRODUCT OF OPERATORS

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*Abstract.* Let  $\mathcal{H}$  and  $\mathcal{K}$  be two infinite-dimensional complex Hilbert spaces. Let  $\mathcal{B}(\mathcal{H})$  denote the algebra of all bounded linear operators on  $\mathcal{H}$ . If  $T$  is an operator in  $\mathcal{B}(\mathcal{H})$  and  $x$  a vector in  $\mathcal{H}$  then  $\gamma_T(x)$  denotes the peripheral local spectrum of  $T$  at  $x$ . In this paper we characterize all surjective maps  $\varphi$  from  $\mathcal{B}(\mathcal{H})$  onto  $\mathcal{B}(\mathcal{K})$  satisfying

$$\gamma_{(\mu ST^*S + \nu T^*S)}(h_0) = \gamma_{(\mu\varphi(S)\varphi(T)^*\varphi(S) + \nu\varphi(T)^*\varphi(S))}(k_0), \quad (S, T \in \mathcal{B}(\mathcal{H})),$$

for a given couple of complex scalars  $(\mu, \nu) \neq (0, 0)$  and nonzero vectors  $h_0 \in \mathcal{H}$  and  $k_0 \in \mathcal{K}$ . This result provides a complete description of all surjective maps from  $\mathcal{B}(\mathcal{H})$  onto  $\mathcal{B}(\mathcal{K})$  preserving the peripheral local spectrum of the skew double product " $T^*S$ " and the skew triple product " $TS^*T$ " of operators. It also unifies and extends several known results on local spectrum preservers.

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### REFERENCES

- [1] Z. ABDELALI, A. ACHCHI AND R. MARZOUKI, *Maps preserving the local spectrum of skew-product of operators*, Linear Algebra and its Applications, **485**, (2015), 58–71.
- [2] Z. ABDELALI, A. ACHCHI AND R. MARZOUKI, *Maps preserving the local spectrum of some matrix products*, Operators And Matrices, (2018) 549–562.
- [3] P. AIENA, *Fredholm and Local Spectral Theory, with Applications to Multipliers*, Kluwer, Dordrecht, (2004).
- [4] G. AN AND J. HOU, *Rank-preserving multiplicative maps on  $\mathcal{B}(X)$* , Linear Algebra and its Applications, **342**, (2002), 59–78.
- [5] M. BENDAOU, *Preservers of local spectrum of matrix Jordan triple products*, Linear Algebra and its Applications, **471**, (2015), 604–614.
- [6] M. BENDAOU, M. JABBAR AND M. SARIH, *Preservers of local spectra of operator products*, Linear and Multilinear Algebra, **63**(4), (2015), 806–819.
- [7] R. BHATIA, P. ŠEMRL AND A. SOUOUR, *Maps on matrices that preserve the spectral radius distance*, Studia Mathematica, **134**(2), (1999), 99–110.
- [8] A. BOURHIM, T. JARI AND J. MASHREGHI, *Peripheral local spectrum preservers and maps increasing the local spectral radius*, Operators and Matrices, **1269**, (2016), 189–208.
- [9] A. BOURHIM AND M. MABROUK, *Maps preserving the local spectrum of Jordan product of matrices*, Linear Algebra and its Applications, **484**, (2015), 379–395.
- [10] A. BOURHIM AND J. MASHREGHI, *Local spectral radius preservers*, Integral Equations and Operator Theory, **76**(1), (2013), 95–104.
- [11] A. BOURHIM AND J. MASHREGHI, *Maps preserving the local spectrum of product of operators*, Glasgow Math. J, (2015), 709–718.
- [12] A. BOURHIM AND J. MASHREGHI, *Maps preserving the local spectrum of triple product of operators*, Linear and Multilinear Algebra, **63**(4), (2015), 765–773.
- [13] A. BOURHIM AND J. MASHREGHI, *A survey on preservers of spectra and local spectra*, Amer. Math. Soc., Providence, RI, (2015), 45–98.

- [14] A. BOURHIM AND V. G. MILLER, *Linear maps on  $\mathcal{M}_n(\mathbb{C})$  preserving the local spectral radius*, *Studia Mathematica*, **188**(1), (2008), 67–75.
- [15] A. BOURHIM AND T. RANSFORD, *Additive maps preserving local spectrum*, *Integral Equations Operator Theory*, **55**, (2006), 377–385.
- [16] J. T. CHAN, C. K. LI AND N. S. SZE, *Mappings preserving spectra of products of matrices*, *Amer. Math. Soc.*, **135**, (2007), 977–986.
- [17] C. COSTARA, *Linear maps preserving operators of local spectral radius zero*, *Integral Equations and Operator Theory*, **73**(1), (2012), 7–16.
- [18] J. L. CUI AND J. C. HOU, *Maps leaving functional values of operator products invariant*, *Linear Algebra and its Applications*, **428**, (2008), 1649–1663.
- [19] J. L. CUI AND C. K. LI, *Maps preserving peripheral spectrum of Jordan products of operators*, *Operators and Matrices*, **6**, (2012), 129–146.
- [20] M. DOLLINGER AND K. OBERAI, *Variation of local spectra*, *J. Math. Anal. Appl.* **39** (1972) 324–337.
- [21] M. GONZÁLEZ AND M. MBEKHTA, *Linear maps on  $M_n(\mathbb{C})$  preserving the local spectrum*, *Linear Algebra and its Applications*, **427**, (2007), 176–182.
- [22] J. C. HOU AND Q. H. DI, *Maps preserving numerical range of operator products*, *Amer. Math. Soc.*, **134**, (2006), 1435–1446.
- [23] T. MIURA AND D. HONMA, *A generalization of peripherally-multiplicative surjections between standard operator algebras*, *Cent. Eur. J. Math.*, **7**(3), (2009), 479–486.
- [24] L. MOLNÁR, *Some characterizations of the automorphisms of  $B(H)$  and  $C(X)$* , *Amer. Math. Soc.*, **130**(1), (2002), 111–120.
- [25] M. OLMADIĆ AND P. ŠEMRL, *Additive mappings preserving operators of rank one*, *Linear Algebra and its Applications*, **182**, (1993), 239–256.
- [26] K. B. LAURSEN AND M. M. NEUMANN, *An introduction to local spectral theory*, *London Mathematical Society Monograph, New Series*, vol. 20, 2000.
- [27] C. K. LI, P. ŠEMRL AND N. S. SZE, *Maps preserving the nilpotency of products of operators*, *Linear Algebra and its Applications*, **424**, (2007), 222–239.
- [28] M. WANG, L. FANG AND G. JI, *Linear maps preserving idempotency of products or triple Jordan products of operators*, *Linear Algebra and its Applications*, **429**, (2008), 181–189.
- [29] W. ZHANG AND J. HOU, *Maps preserving peripheral spectrum of Jordan semi-triple products of operators*, *Linear Algebra and its Applications*, **435**, (2011), 1326–1335.