

## ON REAL OR INTEGRAL SKEW LAPLACIAN SPECTRUM OF DIGRAPHS

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*Abstract.* For a simple connected graph  $G$  with  $n$  vertices and  $m$  edges, let  $\vec{G}$  be a digraph obtained by giving an arbitrary direction to the edges of  $G$ . In this paper, we consider the skew Laplacian matrix of a digraph  $\vec{G}$  and we obtain the skew Laplacian spectrum of the orientations of a complete bipartite graph, complete split graph and the join of two graphs. We prove that deleting an edge of a Hamiltonian path in a transitive tournament does not effect the skew Laplacian spectrum. We show the existence of various families of skew Laplacian integral digraphs.

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