

## RANKS OF COMMUTATORS OF TRUNCATED TOEPLITZ OPERATORS ON FINITE DIMENSIONAL SPACES

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**Abstract.** We study the rank of commutator  $[A_\eta, A_\eta^*]$  of truncated Toeplitz operators  $A_\eta$  and  $A_\eta^*$  with several type of inner symbols  $\eta$  on the model space  $\mathcal{H}_\theta$  with finite Blaschke product  $\theta$ .

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