

THE OUTER PRODUCT AND OUTER PRODUCT SPACE

THEOPHILUS AGAMA

Abstract. In this note we introduce the notion of the outer product of elements in a vector space. We study their properties and explore their applications. In particular, we show that under certain conditions the inequality holds

$$\sum_{\lambda_i \in \text{Spec}(ab^T)} \min\{\log|t - \lambda_i|\}_{\|a\|, \|b\|} \leq \#\text{Spec}(ab^T) \log\left(\frac{\|b\| + \|a\|}{2}\right) + \frac{1}{\|b\| - \|a\|} \sum_{\lambda_i \in \text{Spec}(ab^T)} \log\left(1 - \frac{2\lambda_i}{\|b\| + \|a\|}\right).$$

Also under the same condition, the inequality also holds

$$\int_{\|a\|}^{\|b\|} \log|\det(ab^T - tI)| dt \leq \#\text{Spec}(ab^T)(\|b\| - \|a\|) \log\left(\frac{\|b\| + \|a\|}{2}\right) + \sum_{\lambda_i \in \text{Spec}(ab^T)} \log\left(1 - \frac{2\lambda_i}{\|b\| + \|a\|}\right).$$

Mathematics subject classification (2020): Primary 54C40, 14E20; Secondary 46E25, 20C20.

Keywords and phrases: Outer product, norm.

REFERENCES

- [1] W. RUDIN, *Real and complex analysis*, Tata McGraw-hill education, 2006.