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# NEW CHARACTERIZATIONS FOR DIFFERENCES OF INTEGRAL–TYPE OPERATORS FROM $\alpha$–BLOCH SPACE TO $\beta$–BLOCH–ORLICZ SPACE

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*Abstract.* Several new equivalent characterizations for the boundedness of the differences of integral-type operators from $\alpha$-Bloch space to $\beta$-Bloch-Orlicz space are presented in this paper. Especially, we estimate their essential norms in terms of the $n$-th power of the induced analytic self-maps on the unit disk, which can provide new and interesting compactness criteria and be seen as extensions of several existing results in the literature. Moreover, some applications are also exhibited in an example for readers' convenience.

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