

ANALYTIC EXTENSION OF  $n$ -NORMAL OPERATORS

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**Abstract.** Normal operators and  $n$ -normal operators played a pivotal role in the development of operator theory. In order to generalize these classes of operators, we introduce new classes of operators which we call analytic extension of  $n$ -normal operator and  $F$ -quasi- $n$ -normal operator. We show that every analytic extension of  $n$ -normal operator and  $F$ -quasi- $n$ -normal operator have scalar extensions. We also show that an analytic extension of  $n$ -normal operator has a nontrivial invariant subspace. Some spectral properties are also presented.

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