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AN OPPENHEIM TYPE DETERMINANTAL INEQUALITY FOR THE KHATRI–RAO PRODUCT

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Abstract. The Khatri-Rao product is a generalization of the classical Hadamard product for block matrices. In this paper, we give an Oppenheim type determinantal inequality for the Khatri-Rao product of two block positive semidefinite matrices, and then we extend our result to multiple block matrices.

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