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EXTENSION OF GENERALIZED STRONG DRAZIN INVERSE

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Abstract. As an extension of the generalized strong Drazin inverse, we present a new generalized inverse for Banach algebra elements based on a g–Drazin invertible element rather than on a quasinilpotent element in the definition of the generalized strong Drazin inverse. Because of that, our new inverse will be called an extended gs–Drazin inverse. Some characterizations of this inverse are given using idempotents and tripotents. We also study extensions of Cline's formula to the case of extended gs–Drazin inverse. Applying these results, we introduce and investigate an extended s–Drazin inverse.

Mathematics subject classification (2020): 46H05, 46H99, 15A09.
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