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MINIMAL FAMILIES OF LIMIT OPERATORS

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Abstract. We study two abstract scenarios, where an operator family has a certain minimality property. In both scenarios, it is shown that norm, spectrum and resolvent are the same for all family members. Both abstract settings are illustrated by practically relevant examples, including discrete Schrödinger operators with periodic, quasiperiodic, almost-periodic, Sturmian and pseudo-ergodic potential. The main tool is the method of limit operators, known from studies of Fredholm operators and convergence of projection methods. We close by connecting this tool to the study of subwords of the operator potential.

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