

SOME RESULTS ON CAUCHY DUAL OF CONDITIONAL TYPE OPERATORS

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Abstract. In this paper, we discuss measure theoretic some characterizations of the notion of Cauchy dual for Lambert conditional operators in some operator classes on $L^2(\Sigma)$ such as, n -normal, n -quasi-normal, n -power \dagger -normal, n -power \dagger -quasi-normal. Moreover, the relations between these classes and some basic properties of these operators are studied. Finally, using the matrix representation, some examples are provided to illustrate the obtained results.

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