

## SOME REFINEMENTS OF REAL POWER FORM INEQUALITIES FOR CONVEX FUNCTIONS VIA WEAK SUB-MAJORIZATION

MOHAMED AMINE IGHACHANE\* AND MOHAMMED BOUCHANGOUR

*Abstract.* The main goal of this article, is to develop a general method for improving some new real power inequalities for convex and log-convex functions, which extends and unifies two recent and important results due to M. Sababheh [Linear Algebra Appl. **506** (2016), 588–602] and D. Q. Huy et al. [Linear Algebra Appl. **656** (2023), 368–384]. Then by selecting some appropriate convex and log-convex functions, we obtain new mean inequalities for scalars and matrices, some new refinements and reverses of the Heinz and Hölder type inequalities for matrices. We get also some new and refined trace and numerical radius inequalities.

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