

A GENERALIZATION OF THE WEIGHTED ALGEBRAIC NUMERICAL RADIUS ON C^* -ALGEBRAS

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Abstract. Let $N(\cdot)$ be a norm on a unital C^* -algebra \mathfrak{A} . For s and t are both nonnegative reals and $s+t > 0$, we introduce a family of non-negative real-valued functions on \mathfrak{A} , defined by

$$v_{(N,(s,t))}(x) = \sup_{\theta \in \mathbb{R}} N\left(\Re_{(s,t)}\left(e^{i\theta}x\right)\right), \quad (x \in \mathfrak{A}).$$

Here, $\Re_{(s,t)}(e^{i\theta}x) = se^{i\theta}x + t(e^{i\theta}x)^*$ for all $x \in \mathfrak{A}$. Some basic properties and other useful characterizations of this family of functions are presented. As a special case of this family of functions, some results involving the weighted algebraic numerical radius are obtained. Additionally, we establish the equivalence between the numerical radius $v(x)$ and the norm $v_{(s,t)}(x)$.

Mathematics subject classification (2020): Primary 47A30; Secondary 47B10, 46B20.

Keywords and phrases: C^* -algebra, weighted algebraic numerical radius, C^* -norm.

REFERENCES

- [1] A. ABU-OMAR, F. KITTANEH, *A generalization of the numerical radius*, Linear Algebra Appl. **569**, 323–334 (2019).
- [2] A. ALAHMARI, M. MABROUK, A. ZAMANI, *Further results on the a -numerical range in C^* -algebras*, Banach Journal of Mathematical Analysis **16** (2), 25 (2022).
- [3] N. ALTWAIJRY, K. FEKI, N. MINCULETE, *On some generalizations of Cauchy-Schwarz inequalities and their applications*, Symmetry **15** (2), 304 (2023).
- [4] H. BAKLOUTI, K. FEKI, O. A. M. SID AHMED, *Joint numerical ranges of operators in semi-Hilbertian spaces*, Linear Algebra Appl. **555**, 266–284 (2018).
- [5] P. BHUNIA, S. BAG, K. PAUL, *Bounds for zeros of a polynomial using numerical radius of Hilbert space operators*, Ann. Funct. Anal. **12** (2), 21 (2021).
- [6] P. BHUNIA, S. BAG, K. PAUL, *Numerical radius inequalities and its applications in estimation of zeros of polynomials*, Linear Algebra Appl. **573**, 166–177 (2019).
- [7] P. BHUNIA, K. PAUL, *Furtherance of numerical radius inequalities of Hilbert space operators*, Archiv der Mathematik **117** (5), 537–546 (2021).
- [8] F. BONSALL, J. DUNCAN, *Numerical Range II*, London Mathematical Society Lecture Note Series, Cambridge University Press, Cambridge (1973).
- [9] A. BOURHIM, M. MABROUK, *a -numerical range on C^* -algebras*, Positivity **25** (4), 1489–1510 (2021).
- [10] A. BOURHIM, M. MABROUK, *Numerical radius and product of elements in C^* -algebras*, Linear and Multilinear Algebra **65** (6), 1108–1116 (2017).
- [11] C. CONDE, K. FEKI, *Some numerical radius inequality for several semi-Hilbert space operators*, Linear and Multilinear Algebra **71** (6), 1054–1071 (2023).
- [12] S. S. DRAGOMIR, *Power inequalities for the numerical radius of a product of two operators in Hilbert spaces*, Sarajevo J. Math. **5** (18), 269–278 (2009).
- [13] K. FEKI, F. KITTANEH, *Some new refinements of generalized numerical radius inequalities for Hilbert space operators*, Mediterr. J. Math. **19** (1), 17 (2022).
- [14] F. GAO, X. LIU, *Inequalities for the weighted A -numerical radius of semi-Hilbertian space operators*, Oper. Matrices **17** (2), 343–354 (2023).

- [15] F. KITTANEH, *Numerical radius inequalities for Hilbert space operators*, Stud. Math. **168** (1), 73–80 (2005).
- [16] M. MABROUK, A. ZAMANI, *An extension of the a -numerical radius on C^* -algebras*, Banach Journal of Mathematical Analysis **17** (3), 42 (2023).
- [17] G. J. MURPHY, *C^* -algebras and Operator Theory*, Academic Press, New York, (1990).
- [18] M. E. OMIÐVAR, H. R. MORADI, *Better bounds on the numerical radius of Hilbert space operators*, Linear Algebra Appl. **604**, 265–277 (2020).
- [19] M. E. OMIÐVAR, H. R. MORADI, *New estimates for the numerical radius of Hilbert space operators*, Linear and Multilinear Algebra **69** (5), 946–956 (2021).
- [20] N. C. ROUT, S. SAHOO, D. MISHRA, *Some A -numerical radius inequalities for semi-Hilbertian space operators*, Linear Multilinear Algebra **69** (5), 980–996 (2021).
- [21] S. SAHOO, N. C. ROUT, *New upper bounds for the numerical radius of operators on Hilbert spaces*, Adv. Oper. Theory **7** (4), 50 (2022).
- [22] A. SHEIKHHOSSEINI, M. KHOSRAVI, M. SABABHEH, *The weighted numerical radius*, Ann. Funct. Anal. **13** (1), 3 (2022).
- [23] A. ZAMANI, *Characterization of numerical radius parallelism in C^* -algebras*, Positivity **23** (2), 397–411 (2019).
- [24] A. ZAMANI, P. WÓJCIK, *Numerical radius orthogonality in C^* -algebras*, Ann. Funct. Anal. **11** (4), 1081–1092 (2020).
- [25] A. ZAMANI, *The weighted Hilbert-Schmidt numerical radius*, Linear Algebra Appl. **675**, 225–243 (2023).