

JACOBSON'S LEMMA FOR THE OPERATORS ADMITTING SAPHAR DECOMPOSITIONS

MIRJANA D. ŠIKULJAK

Abstract. We extend Jacobson's lemma to the operators of Saphar type and the operators admitting the generalized Saphar decomposition and the generalized Saphar-Riesz decomposition.

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