Operators and Matrices
Volume 19, Number 1 (2025), 85–100
doi:10.7153/oam-2025-19-06

OPERATOR RADII OF COMMUTING PRODUCTS

AMIT KUMAR, KHOLE TIMOTHY POUMAI AND S. K. KAUSHIK

Abstract. Let $\omega_\rho(X)$ represent the operator radius of a bounded linear operator X on a Hilbert space $\mathscr{H}$, where $0<\rho\leqslant 2$ and let X and Y be bounded commutative operators on $\mathscr{H}$. In this article, the long-standing constant $k=1.169$ in the inequality $\omega_2(XY)\leqslant 1.169\omega_1(X)\omega_2(Y)$ is improved and more accurate estimates for the ratio $\omega_\rho(XY)/(\omega_1(X)\cdot\omega_\rho(Y))$ are obtained.

Mathematics subject classification (2020): 47A20, 47A30, 47A10, 47A12.

Keywords and phrases: Operator radius, numerical radius, contraction, unitary ρ dilation, commutative operators, Hilbert space.

REFERENCES

[1] KENNETH R. DAVIDSON AND JOHN A. R. HOLBROOK, *Numerical radii of zero-one matrices*, Michigan Math. J., no. 2, **35** (1988), 261–267.
[2] JOHN A. R. HOLBROOK, *Inequalities governing the operator radii associated with unitary ρ-dilations*, Michigan Math. J., **18** (1971), 149–159.
[3] JOHN A. R. HOLBROOK, *Multiplicative properties of the numerical radius in operator theory*, J. Reine Angew. Math., **237** (1969), 166–174.
[4] JOHN A. R. HOLBROOK, *On the power-bounded operators of Sz.-Nagy and Foiaş*, Acta Sci. Math. (Szeged), **29** (1968), 299–310.
[5] JOHN HOLBROOK AND JEAN-PIERRE SCHOCH, *Theory vs. experiment: multiplicative inequalities for the numerical radius of commuting matrices*, Oper. Theory Adv. Appl., **202** (2010), 273–284.
[6] FUAD KITTANEH AND ALI ZAMANI, *On the ρ-operator radii*, Linear Algebra Appl., **687** (2024), 132–156.
[7] VLADIMÍR MÜLLER, *The numerical radius of a commuting product*, Michigan Math. J., no. 2, **35** (1988), 255–260.
[8] K. OKUBO AND T. ANDO, *Operator radii of commuting products*, Proc. Amer. Math. Soc., **56** (1976), 203–210.
[9] BÉLA SZ.-NAGY, *Products of operators of classes C_ρ*, Rev. Roumaine Math. Pures Appl., **21** (1987), 13(1968), 897–899.
[10] BÉLA SZ.-NAGY AND CIPRIAN FOIAŞ, *On certain classes of power-bounded operators in Hilbert space*, Acta Sci. Math. (Szeged), **27** (1966), 17–25.
[11] H. S. WALL, *Analytic Theory of Continued Fractions*, D. Van Nostrand Co., Inc., New York, 1948, https://doi.org/10.1007/978-1-4612-0653-8.
[12] JAMES P. WILLIAMS, *Schwarz norms for operators*, Pacific J. Math., **24** (1968), 181–188.