

STRONG COMMUTATIVITY PRESERVING ADDITIVE MAPS ON INVERTIBLE TRIANGULAR MATRICES OVER \mathbb{F}_2

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Abstract. Let $T_n(\mathbb{F}_2)$ be the ring of $n \times n$ upper triangular matrices over the Galois field \mathbb{F}_2 of two elements. In this paper we characterize strong commutativity preserving additive maps $\psi : T_n(\mathbb{F}_2) \rightarrow T_n(\mathbb{F}_2)$ on invertible matrices for $n = 2$ and $n \geq 5$. This result completes a recent result obtained by Chooi et al. in [14] and yields a comprehensive structural characterization of strong commutativity preserving additive maps on rank k upper triangular matrices over division rings. Some irregular forms are included to exemplify the complexity in structure of strong commutativity preserving additive maps $\psi : T_n(\mathbb{F}_2) \rightarrow T_n(\mathbb{F}_2)$ on invertible matrices for $n = 3$ and 4.

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