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EXISTENCE OF MILD SOLUTION FOR IMPULSIVE STOCHASTIC DIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITIONS

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Abstract. This paper is concerned with the existence of mild solution for impulsive stochastic differential equations with nonlocal conditions in PC-norm. Our approach is based on Krasnoselskii fixed point theorem.

1. Introduction

Stochastic differential equation is an emerging field drawing attention from both theoretical and applied disciplines, which has been successfully applied to problems in mechanical, electrical, economics, physics and several fields in engineering. For details, see [10, 16] and the references therein. Recently a large number of interesting results of stochastic equations have been reported in [5, 8, 11, 15, 20, 22, 23]. For example, in [22], Taniguchi et al. discussed the existence, uniqueness, and asymptotic behavior of mild solutions to stochastic partial functional differential equations with finite delay. Xu et al. [23] established stochastic versions of the well-known Picard local existence-uniqueness theorem and continuation theorem given by Hale [7] for functional differential equations.

The nonlocal Cauchy problem was first introduced by Byszewski and Lakshmikantham [4]. Since it is demonstrated that the nonlocal problem have better effects in applications than the classical ones, so differential equations with nonlocal problem have been studied extensively in the literatures. For more details on this topic we refer to [1, 3, 9, 17, 18, 19] and references therein.

Recently, there are some results of mild solutions for stochastic differential equation with nonlocal conditions. For example, by using Leray-Schauder fixed point approach, the existence of mild solutions for semilinear stochastic delay evolution equation with nonlocal conditions was studied respectively in [3]. In [2], Balasubramaniam et al. discussed the existence of mild and strong solutions of semilinear neutral functional differential evolution equations with nonlocal conditions by using fractional power of operators and Krasnoselskii fixed point theorem.

Impulsive effects are common phenomena due to instantaneous perturbations at certain moment, such phenomena are described by impulsive differential equation which

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have been used efficiently in modelling many practical problems that arise in the fields of engineering, physics, and science as well. So the theory of impulsive differential equations is also attracting much attention in recent years [13, 21]. Correspondingly, a lot of existence of mild solution for impulsive differential equations with nonlocal conditions have been obtained in [6, 14]. In [20], the authors studied the existence and asymptotic stability in pth moment of mild solutions to nonlinear impulsive stochastic differential equation.

To the best of our knowledge, there is no work reported on impulsive stochastic differential equations with nonlocal conditions. Motivated by the above works, the purpose of this paper is to prove the existence and uniqueness of mild solutions for the impulsive stochastic differential equations with nonlocal conditions in PC-norm. Our approach is based on the fixed point theorem. The rest of this paper is organized as follows. In Section 2, impulsive stochastic differential equations are presented, together with definition of mild solution. Finally in Section 3, the existence results on mild solutions are derived.

2. Preliminaries

Let H be a real separable Hilbert spaces with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|$, and let K be another real separable Hilbert spaces with inner product $(\cdot,\cdot)_K$ and norm $\|\cdot\|_K$. $L(K,H)$ denotes the space of bounded operators from K to H. Let $(\Omega,\mathscr{F},P)$ be a complete probability space equipped with a complete family of right continuous increasing sub σ-algebras $\{\mathscr{F}_t, t\geqslant 0\}$ satisfying $\mathscr{F}_t\subset\mathscr{F}$.

Let $\beta_n(t), n=1,2,\dots$ be a sequence of real-valued one-dimensional standard Brownian motions mutually independent over $(\Omega,\mathscr{F},P)$. Set

$$w(t)=\sum_{n=1}^{\infty}\sqrt{\lambda_n}\beta_n(t)\xi_n, t\geqslant 0,$$

where $\lambda_n\geqslant 0$, $(n=1,2,\dots)$ are nonnegative real numbers and $\{\xi_n\}(n=1,2,\dots)$ is a complete orthonormal basis in K, Let $Q\in L(K,K)$ be an operator defined by

$$Q\xi_n=\lambda_n\xi_n \quad \text{with a finite trace } Tr(Q)=\sum_{n=1}^{\infty}\lambda_n<\infty.$$

Then, the above K-valued stochastic process $\omega(t)$ is called a Q-Wiener process.

Let $\varphi\in L(K,H)$ and define

$$\|\varphi\|_{L_2^0}^2=Tr(\varphi Q\varphi^*)=\Big\{\sum_{n=1}^{\infty}\|\sqrt{\lambda_n}\varphi\xi_n\|^2\Big\}.$$

If $\|\varphi\|_{L_2^0}<\infty$, then φ is called a Q-Hilbert-Schmidt operator, where $L_2^0(K,H)$ denote the space of all Q-Hilbert-Schmidt operators $\varphi: K\to H$.

We denote $L^p(\Omega,H)$ the collection of all strongly-measurable, p-integrable H-valued variables with norm

$$\|x(\cdot)\|_{L^p}=\Big(E\|x(\cdot;\omega)\|_H^p\Big)^{\frac{1}{p}},$$

where E is defined by $E(h)=\int_\Omega h(\omega)dP$. Let $\mathscr{C}=\mathscr{C}(J,L^p(\Omega,H))$ be the Banach space of all continuous maps from J into $L^p(\Omega,H)$ satisfying $\sup\limits_{t\in J}E\|x(t)\|^p<\infty$. Let

$$PC(J,L^p(\Omega,H))=\big\{\psi:J\to L^p(\Omega,H)\;|\,\psi\in\mathscr{C}((t_k,t_{k+1}],H),\;k=0,1,\dots,q,\\ \psi(t_k^+),\psi(t_k^-)\text{ exist and }\psi(t_k^-)=\psi(t_k)\big\}$$

with the norm

$$\|\psi\|_{PC}=\sup_{t\in J}(\|\psi(t)\|_{L^p}^p)^{\frac{1}{p}}.$$

Then $(PC(J,L^p(\Omega,H)),\|\cdot\|_{PC})$ is a Banach space.

In this paper, we consider the existence of mild solution for the following impulsive stochastic differential equations in a Hilbert space

$$\begin{cases} dx(t)=[Ax(t)+F(t,x(t),x(a_1(t)),\dots,x(a_v(t)))]dt\\ \qquad\qquad +G(t,x(t),x(b_1(t)),\dots,x(b_m(t)))d\omega(t),\; t\in J=[0,T],\; t\neq t_k,\\ \triangle x(t_k)=I_k(x(t_k)),\; k=1,2,\dots,q,\\ x(0)=x_0+g(x), \end{cases}\tag{1}$$

where $x_0\in H$, $a_i,b_j:J\to J, i=1,2,\dots,v, j=1,2,\dots,m$ are continuous, A is the infinitesimal generator of an analytic semigroup of bounded linear operators $S(t),t\geqslant 0$,

$$x(t_k^+)=\lim_{h\to 0+}x(t_k+h)\quad\text{and}\quad x(t_k)=\lim_{h\to 0-}x(t_k+h).$$

The $t_k\geqslant 0$ are impulsive moments satisfying

$$t_k<t_{k+1}\quad\text{and}\quad \lim_{k\to+\infty}t_k=+\infty.$$

The $\triangle x(t_k)=x(t_k^+)-x(t_k)$ represents the jump in the state x at t_k.

We assume that $\|S(t)\|\leqslant M$ for $t\in T$ and $0\in\rho(A)$, where $\rho(A)$ is the resolvent set of A. Let us recall the mild solution for nonlocal Cauchy problem (1) as follows.

DEFINITION 2.1. A stochastic process $x(t)\in PC(J,L^p(\Omega,H))$ is called a mild solution for nonlocal Cauchy problem (1), if:

(i) $x(t)$ is adapted to $\mathscr{F}_t$;

(ii) $x(t)\in H$ has a càdlàg path on $t\in[0,T]$ almost surely;

(iii) for arbitrary $t\in[0,T]$,

$$\begin{aligned} x(t)=S(t)[x_0+g(x)]&+\int_0^t S(t-s)F(s,x(s),x(a_1(s)),\dots,x(a_v(s)))ds\\ &+\int_0^t S(t-s)G(s,x(s),x(b_1(s)),\dots,x(b_m(s)))d\omega(s)\\ &+\sum_{0<t_k<t}S(t-t_k)I_k(x(t_k)). \end{aligned}\tag{2}$$

LEMMA 2.1. *(see [5]) For any $r \geqslant 1$ and for arbitrary $L_2^0(K,H)$-valued predictable process $\Phi(\cdot)$*

$$\sup_{s\in[0,t]} E\Big\|\int_0^s \Phi(u)d\omega(u)\Big\|^{2r} \leqslant C_r\Big(\int_0^t \Big(E\|\Phi(s)\|_{L_2^0}^{2r}\Big)^{\frac{1}{r}} ds\Big)^r,\ t \geqslant 0, \tag{3}$$

where $C_r = (r(2r-1))^r$.

Furthermore, one imposes the following important assumptions to obtain the existence of nonlocal Cauchy problem (1):

(H_1) $F : J\times H^{v+1} \to H$ is a continuous function, and there exist $L_F, \overline{L}_F > 0$ such that for $0 \leqslant s_1,s_2 \leqslant T, x_i,y_i \in H, i = 0,1,\dots,v$

$$\|F(s_1,x_0,x_1,\dots,x_v) - F(s_2,y_0,y_1,\dots,y_v)\|^p \leqslant L_F\Big(\|s_1-s_2\|^p + \max_{i=0,1,\dots,v}\|x_i-y_i\|^p\Big)$$

and for $(t,x_0,x_1,\dots,x_v) \in J\times H^{v+1}$

$$\|F(t,x_0,x_1,\dots,x_v)\| \leqslant \overline{L}_F\Big(\max_{i=0,1,\dots,v}\|x_i\|^p + 1\Big);$$

(H_2) the function $G : J\times H^{m+1} \to L(K,H)$ satisfies the following conditions:

(i) for each $t \in J$, $G(t,\cdot) : H^{m+1} \to L(K,H)$ is continuous and for $(x_0,x_1\dots,x_m) \in H^{m+1}$, $G(\cdot,x_0,x_1,\dots,x_m) : J \to L(K,H)$ is $\mathscr{F}_t$-measurable;

(ii) for any $l > 0$, there exists a function $\rho_l \in L^1(J)$ such that

$$\sup_{\|x_0\|^p,\dots,\|x_m\|^p\leqslant l} E\|G(t,x_0,x_1,\dots,x_m)\|_{L_2^0}^p \leqslant \rho_l(t)$$

and

$$\liminf_{l\to+\infty}\frac{1}{l}\Big[\int_0^T \rho_l(s)^{\frac{2}{p}}ds\Big]^{\frac{p}{2}} = \eta < \infty;$$

(H_3) the function $G : J\times H^{n+1} \to L(K,H)$ satisfies $(H_2)(i)$, and there exists $L_G > 0$ such that for $0 \leqslant s_1,s_2 \leqslant T,\ x_i,y_i \in H,\ i = 0,1,\dots,m$,

$$\|G(s_1,x_0,x_1,\dots,x_m) - G(s_2,y_0,y_1,\dots,y_m)\|_{L_2^0}^p \leqslant L_G\Big(\|s_1-s_2\|^p + \max_{i=0,1,\dots,m}\|x_i-y_i\|^p\Big);$$

(H_4) $g : H \to L_2^0(\Omega,H)$ is completely continuous and there exists a nondecreasing function $N : R^+ \to R^+$ such that for all $x \in H$,

$$\|g(x)\|^p \leqslant N(\|x\|^p) \quad\text{and}\quad \liminf_{l\to+\infty}\frac{N(l)}{l} = \delta < \infty;$$

(H_5) $g : H \to L_2^0(\Omega,H)$, and there exists $L_g \geqslant 0$ such that for any $x,y \in H$

$$\|g(x)-g(y)\|^p \leqslant L_g\|x-y\|^p;$$

(H_6) $I_k : H \to H, k = 1, \ldots, q$, and there exists $h_k \geqslant 0$ such that for any $x, y \in H$

$$\|I_k(x) - I_k(y)\|^p \leqslant h_k \|x - y\|^p \quad \text{and} \quad \|I_k(x)\|^p \leqslant \overline{h}_k \|x\|^p.$$

Our main results are based on the following Krasnoselskii fixed point theorem [12].

LEMMA 2.2. *Let B be a closed convex and nonempty subset of a Banach space X. Let L and N be be two operators such that*
(i) $Lx + Ny \in B$ whenever $x, y \in B$;
(ii) L is a contraction mapping;
(iii) N is compact and continuous.
Then there exists $z \in B$ such that $z = Lz + Nz$.

3. Main results

In this section, we present and prove our main results.

THEOREM 3.1. *Assume that $(H_1), (H_2), (H_4)$ and (H_6) hold and $x(0) \in L_2^0$, then the nonlocal problem (1) has a mild solution provided that*

$$4^{p-1} M^p \left(\delta + \overline{L}_F T^p + C_p \eta + \sum_{k=1}^{q} \overline{h}_k \right) < 1 \tag{4}$$

and

$$2^{p-1} M^p \left(L_F T^p + \sum_{i=1}^{q} h_k \right) < 1. \tag{5}$$

Proof. Define the map $\Phi : PC(J, L^p(J, H)) \to PC(J, L^p(J, H))$ by

$$\begin{aligned}
(\Phi x)(t) = S(t)[x_0 + g(x)] &+ \int_0^t S(t-s) F(s, x(s), x(a_1(s)), \ldots, x(a_v(s))) ds \\
&+ \int_0^t S(t-s) G(s, x(s), x(b_1(s)), \ldots, x(b_m(s))) d\omega(s) \\
&+ \sum_{0 < t_k < t} S(t - t_k) I_k(x(t_k)) \\
= \sum_{i=1}^{4} \Theta_i(t) \ \text{ for a. e. } t \in J. &
\end{aligned} \tag{6}$$

To prove the existence of mild solution of (1), it is enough to show that the operator Φ has fixed point in $PC(J, L^p(J, H))$. Let $B_l = \{x \in PC(J, L^p(J, H)) : \|x(t)\|_{PC}^p \leqslant l, t \in J\}$. Then we finish the proof in the following steps.

Step 1. We show that $\Phi B_l \subset B_l$. If it is not true, then there exists $x_l \in B_l$ and $t_l \in J$ such that $\|(\Phi x_l)(t_l)\|^p > l$. However, from (6), we have

$$E\|(\Phi x_l)(t_l)\|^p \leqslant 4^{p-1}\sum_{i=1}^{4} E\|\Theta_i(t_l)\|^p. \tag{7}$$

From (H_1), by H$\ddot{o}$lder inequality, we have

$$\begin{aligned} E\|\Theta_2(t_l)\|^p &\leqslant E\sup_{t\in J}\Big\|\int_0^t S(t-s)F(s,x(s),x(a_1(s)),\ldots,x(a_v(s)))ds\Big\|^p \\ &= E\sup_{t\in J}\Big\|\int_0^t S(t-s)F(s,x(s),x(a_1(s)),\ldots,x(a_v(s)))ds\Big\|^p \\ &\leqslant T^{p-1}E\sup_{t\in J}\int_0^t \|S(t-s)F(s,x(s),x(a_1(s)),\ldots,x(a_v(s)))\|^p ds \\ &\leqslant \overline{L}_F M^p T^p(l+1). \end{aligned} \tag{8}$$

By Lemma 2.1 and (H_2), we obtain

$$\begin{aligned} E\|\Theta_3(t_l)\|^p &= \Big\|\int_0^{t_l} S(t_l-s)G(s,x(s),x(b_1(s)),\ldots,x(b_m(s)))d\omega(s)\Big\|^p \\ &\leqslant C_pM^p\Big[\int_0^{t_l}(E\|G(s,x(s),x(b_1(s)),\ldots,x(b_m(s)))\|^p_{L_2^0})^{\frac{2}{p}}ds\Big]^{\frac{p}{2}} \\ &\leqslant C_pM^p\Big[\int_0^T \rho_l(s)^{\frac{2}{p}}ds\Big]^{\frac{p}{2}}. \end{aligned} \tag{9}$$

Using (H_6), we obtain

$$E\|\Theta_4(t_l)\|^p = E\Big\|\sum_{0<t_k<t} S(t-t_k)I_k(x(t_k))\Big\|^p \leqslant M^p l\sum_{k=1}^{q}\overline{h}_k. \tag{10}$$

Since inequality

$$\|x+y\|^p \leqslant (1+\xi)^{p-1}(\|x\|^p + \frac{\|y\|^p}{\xi^{p-1}}) \tag{11}$$

holds for $\forall\xi > 0, p \geqslant 1, x, y \in H$, it follows from (H_5) that

$$\begin{aligned} E\|\Theta_1(t_l)\|^p &= E\|S(t_l)[x_0+g(x)]\|^p \\ &\leqslant (1+\xi)^{p-1}M^p\Big[\frac{\|x_0\|^p}{\xi^{p-1}} + \|g(x)\|^p\Big] \\ &\leqslant (1+\xi)^{p-1}M^p\Big[\frac{\|x_0\|^p}{\xi^{p-1}} + N(l)\Big]. \end{aligned} \tag{12}$$

Then from (7), (10), and (12), we have for $\forall\xi > 0$,

$$l \leqslant \|(\Phi x_l)(t_l)\|^p$$

$$\leqslant 4^{p-1}\Big\{(1+\xi)^{p-1}M^p\Big[\frac{\|x_0\|^p}{\xi^{p-1}}+N(l)\Big]$$
$$+\overline{L}_F M^p T^p(l+1)+C_p M^p\Big[\int_0^T \rho_l(s)^{\frac{2}{p}}ds\Big]^{\frac{p}{2}}+M^p l\sum_{k=1}^{q}\overline{h}_k\Big\}. \tag{13}$$

Dividing both sides by l and taking the lower limits as $l\to+\infty$, in view of arbitrary property of $\xi>0$, we have

$$4^{p-1}\Big(M^p\delta+\overline{L}_F M^p T^p(l+1)+C_p M^p\eta+M^p\sum_{k=1}^{q}\overline{h}_k\Big)\geqslant 1,$$

which contradicts the expression (4). Thus, for some l, $\Phi B_l\subset B_l$.

Step 2. We decompose $\Phi=\Phi_1+\Phi_2$ as

$$(\Phi_1 x)(t)=\int_0^t S(t-s)F(s,x(s),x(a_1(s)),\ldots,x(a_v(s)))ds+\sum_{0<t_k<t}S(t-t_k)I_k(x(t_k))$$

and

$$(\Phi_2 x)(t)=S(t)[x_0+g(x)]+\int_0^t S(t-s)G(s,x(s),x(b_1(s)),\ldots,x(b_m(s)))d\omega(s),$$

where the operator Φ_1,Φ_2 are defined on B_l. We shall show that Φ_1 is contraction operator while Φ_2 is a compact operator. We firstly prove that Φ_1 satisfies a contraction condition. In fact, for each $t\in J,x,y\in B_l$, we have

$$\begin{aligned}
&E\|(\Phi_1 x)(t)-(\Phi_1 y)(t)\|^p\\
&\leqslant 2^{p-1}\Big\{\sup_{t\in J}E\Big\|\int_0^t S(t-s)[F(s,x(s),x(a_1(s)),\ldots,x(a_v(s)))\\
&\qquad -F(s,y(s),y(a_1(s)),\ldots,y(a_v(s)))]ds\Big\|^p\\
&\qquad +\sup_{t\in J}E\Big\|\sum_{0<t_k<t}S(t-t_k)[I_k(x(t_k))-I_k(y(t_k))]\Big\|^p\\
&\leqslant 2^{p-1}M^p\Big(L_F T^p+\sum_{i=1}^{q}h_k\Big)\sup_{t\in J}E\|x(t)-y(t)\|^p.
\end{aligned} \tag{14}$$

Next, we prove that Φ_2 is compact. Let $\{x_n\}\subset B_l$ with $x_n\to x$ in B_l, then for each $s\in J,x_n\to x$, we have

$$G(s,x_n(s),x_n(b_1(s)),\ldots,x_n(b_m(s)))$$
$$-G(s,x(s),x(b_1(s)),\ldots,x(b_m(s)))\to 0,\ \ n\to\infty, \tag{15}$$

and

$$g(x_n)\to g(x),\ \ n\to\infty. \tag{16}$$

Since

$$E\|G(s,x_n(s),x_n(b_1(s)),\dots,x_n(b_m(s))) \\ - G(s,x(s),x(b_1(s)),\dots,x(b_m(s)))\|_{L_2^0}^p \leqslant 2^{p-1}\rho_l(s) \quad (17)$$

and

$$E\|g(x_n)-g(x)\|^p \leqslant 2^{p-1}N(l), \quad (18)$$

by the dominated convergence theorem, we have

$$E\|\Phi_2 x_n - \Phi_2 x\|^p \to 0, \quad (19)$$

as $n \to \infty$, that is, Φ_2 is continuous. Next, we prove that Φ_2 is an equicontinuous in B_l. Let $x \in B_l, t_1 \geqslant 0$ and $|\varepsilon|$ be sufficiently small, then by Lemma 2.1, we have

$$\begin{aligned} E\|&(\Phi_2 x)(t_1+\varepsilon) - (\Phi_2 x)(t_1)\|^p \\ &\leqslant 2^{p-1}\Big\{\|S(t_1+\varepsilon)-S(t_1)\|^p\|x_0+g(x)\|^p \\ &+ C_p\Big[\int_0^{t_1}\Big(E\|(S(t_1+\varepsilon-s) \\ &\quad - S(t_1-s))G(s,x(s),x(b_1(s)),\dots,x(s),x(b_m(s)))\|^p\Big)^{\frac{2}{p}}ds\Big]^{\frac{p}{2}} \\ &+ C_p\Big[\int_{t_1}^{t_1+\varepsilon}\Big(E\|S(t_1+\varepsilon-s)G(s,x(s),x(b_1(s)),\dots,x(s),x(b_m(s)))\|^p\Big)^{\frac{2}{p}}ds\Big]^{\frac{p}{2}}\Big\}. \end{aligned} \quad (20)$$

Noting that

$$\|G(s,x(s),x(b_1(s)),\dots,x(s),x(b_n(s)))\|_{L_2^0}^p \leqslant h_l(s) \quad \text{and} \quad h_l \in L^1,$$

we conclude that $E\|(\Phi_2 x)(t_1+\varepsilon)-(\Phi_2 x)(t_1)\|^p \to 0$ as $\varepsilon \to 0$. Hence Φ_2 is an equicontinuous in B_l.

Finally, we need to prove that for $0 \leqslant t \leqslant T$, $(\Phi_2 x)(t)$ is relatively compact in B_l. It is easy to see that $(\Phi_2 x)(0)$ is relatively compact in B_l. Fixed by $t \in (0,T]$, for $0<\varepsilon<T$, $x \in B_l$, we define

$$\begin{aligned} (\Phi_2^\varepsilon x)(t) &= S(t)[x_0+g(x)] \\ &\quad + \int_0^{t-\varepsilon} S(t-s)G(s,x(s),x(b_1(s)),\dots,x(b_n(s)))d\omega(s) \\ &= S(t)[x_0+g(x)] \\ &\quad + S(\varepsilon)\int_0^{t-\varepsilon} S(t-\varepsilon-s)G(s,x(s),x(b_1(s)),\dots,x(b_n(s)))d\omega(s). \end{aligned} \quad (21)$$

It follows from the compactness of $S(\varepsilon)(\varepsilon>0)$ that $\{(\Phi_2^\varepsilon x)(t) : x \in B_l\}$ is relatively compact in H for every $\varepsilon \in (0,t)$. Furthermore, for every $x \in B_l$, we have

$$E\|(\Phi_2 x)(t)-(\Phi_2^\varepsilon x)(t)\|^p \leqslant C_p M_T^p \int_{t-\varepsilon}^{t} h_l(s)ds. \quad (22)$$

Consequently, there are relatively compact sets $\{(\Phi_2^\varepsilon x)(t) : x \in B_l\}$ arbitrary close the $(\Phi_2 x)(t)$, which implies that $(\Phi_2 x)(t)$ is also relatively compact in B_l.

Hence Φ_2 is a compact operator by Arzelá-Ascoli theorem. Therefore, by Lemma 2.2, we show that the nonlocal problem (1) has a mild solution.

In the following, we derive the uniqueness of mild solution for the nonlocal problem (1).

THEOREM 3.2. *Assume that $(H_1),(H_3),(H_5)$ and (H_6) hold and $x(0) \in L_2^0$, then the nonlocal problem (1) has a mild solution provided that*

$$4^{p-1}M^p\Big(L_g + L_F T^p + C_p L_G T^{\frac{p}{2}} + \sum_{k=1}^{q} h_k\Big) < 1. \tag{23}$$

Proof. Let $\Phi : PC(J,L^p(J,H)) \to PC(J,L^p(J,H))$ be defined as Theorem 3.1. For $t \in J, x, y \in PC(J,L^p(J,H))$, we have

$$\begin{aligned}
&E\|(\Phi x)(t) - (\Phi y)(t)\|^p \\
&\qquad \leqslant 4^{p-1}\Big\{E\|S(t)[g(x)-g(y)]\|^p \\
&\qquad\quad + E\Big\|\int_0^t S(t-s)[F(s,x(s),x(a_1(s)),\ldots,x(a_v(s))) \\
&\qquad\qquad\qquad - F(s,y(s),y(a_1(s)),\ldots,y(a_v(s)))]ds\Big\|^p \\
&\qquad\quad + E\Big\|\int_0^t S(t-s)[G(s,x(s),x(b_1(s)),\ldots,x(b_m(s))) \\
&\qquad\qquad\qquad - G(s,x(s),y(b_1(s)),\ldots,y(b_m(s)))]d\omega(s)\Big\|^p \\
&\qquad\quad + E\Big\|\sum_{0<t_k<t} S(t-t_k)[I_k(x(t_k)) - I_k(y(t_k))]\Big\|^p\Big\} \\
&\qquad \leqslant 4^{p-1}\Big(M^p L_g + L_F M^p T^p + C_p M^p L_G T^{\frac{p}{2}} + M^p \sum_{k=1}^{q} h_k\Big)E\|x-y\|^p.
\end{aligned} \tag{24}$$

Thus we have

$$\|(\Phi x)(t) - (\Phi y)(t)\|_{PC}^p \leqslant 4^{p-1}M^p\Big(L_g + L_F T^p + C_p L_G T^{\frac{p}{2}} + \sum_{k=1}^{q} h_k\Big)\|x-y\|_{PC}^p. \tag{25}$$

Therefore, Φ is a contraction from (23), which implies that there exists an unique mild solution for nonlocal problem (1).

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