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ASYMPTOTIC OUTPUT CONTROLLABILITY VIA DYNAMIC MATRIX CONTROL

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Abstract. Motivated by industrial applications, we investigate the so-called Dynamic Matrix Control (DMC) strategy for single-input single-output linear continuous-time time-invariant systems. DMC is a type of Model Predictive Control based on the step response model of the process. We show that if the process is governed by a one-dimensional stable dynamical system, then the method drives the output of the sampled system into the desired setpoint as time goes to infinity, that is, the system is asymptotically output controllable with DMC. For two-dimensional systems, sufficient condition on the asymptotic output controllability is given.

1. Introduction

Model Predictive Control (or Receding Horizon Control) designates a wide range of control methods which make an explicit use of a model of the process to obtain the control signal by minimizing an objective function. The common idea is the receding strategy which means that the objective function is minimized by considering also future control actions along the so-called prediction horizon but only the first control signal is applied to the system, then the horizon is displaced towards the future and the next control signal is recalculated. The various MPC algorithms only differ amongst themselves in the model used to represent the process, the noises and the cost function. There are many areas of industrial applications of MPC, for example chemicals, food processing, automotive and aerospace applications, see the survey papers [7, 15]. For further details on Model Predictive Control, see the monographs [3, 11].

In view of the main idea of the Model Predictive Control strategy it is clear that the process model plays an important role in the method. There are many types of models used, one of which is the so-called step response model. If $u(t)$ and $y(t)$ denote the input and output variables, respectively, then the step response model of the system is

$$y(t) = \sum_{i=1}^{\infty} g_i \Delta u(t-i)$$

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where the constants g_i form the sampled output of the system when a step is applied in the control variabe u. For stable systems one may take a truncated response and with this model the control signal can be calculated by minimizing the objective function along the prediction horizon. The step response model is used by the Dynamic Matrix Control algorithm which was developed at the end of the seventies by Cutler and Ramaker, see [5, 6]. It became a successful and widely used technique in industrial and engineering applications, see, e.g., [2, 4, 13]. However, this method got less attention by the mathematical viewpoint, we are aware only a few papers dealing with the algorithm in a pure mathematical way, investigating its stability, see [9, 18]. Hence our motivation in this paper is theoretical, we would like to understand under what conditions does the method drive the output of the system into the desired setpoint. Since this question has not been investigated in detail from the rigorous mathematical point of view, we study the DMC strategy that belongs to the "first generation" of the MPC method, described in several survey papers as "the past of MPC", and leave the rigorous mathematical investigation of more sophisticated strategies as an object of future work.

The aim of the present paper is the analysis of the algorithm of DMC for single-input single-output (SISO) linear continuous-time time-invariant systems. We apply the DMC as a sampled control, i.e., the continuous system is observed at discrete instants, and the control is kept constant along a step. We are interested in the convergence of the method, more precisely, when the algorithm drives the output of the system into the desired setpoint as time goes to infinity, that is, when the system is asymptotically output controllable with DMC. We show that if the underlying system is one-dimensional and it is stable then it is also asymptotically output controllable with DMC (see Theorem 2 and 3). For two-dimensional systems sufficient condition on the asymptotic output controllability will be given (see Theorem 4). The new features of our approach are as follows. On one hand, estimates on the rate of convergence will also be established for both cases. On the other hand, our considerations are self-contained and the techniques used are mainly elementary, making our approach more accessible for a broader audience.

The paper is organized as follows. In Section 2, we give a brief introduction to the basic theory of Dynamic Matrix Control. In Section 3, the method is described when the state-space representation of the underlying system is known. In Sections 4 and 5, asymptotic output controllability of one-dimensional systems is discussed and Section 6 is devoted to the asymptotic output controllability of two-dimensional systems.

2. The step response model and prediction

2.1. Basics of DMC

In this section we briefly summarize the basics of Dynamic Matrix Control, see [3, 11] for details.

Suppose we have a SISO linear discrete-time time-invariant system with input $u(t)$ and output $y(t)$ $(t \in \mathbb{Z})$. The *unit step response* of the system is generated by the *unit* *step control*

$$u(t)=\begin{cases}0, \text{ if } & t\leqslant 0,\\ 1, \text{ if } & t>0\end{cases}$$

assuming (without loss of generality) that $y(0)=0$. Denote $g_i=y(i)$ $(i=0,1,\dots)$, where obviously $g_0=0$, then according to the *step response model* the system is described by the equation

$$y_{\text{model}}(t)=\sum_{i=1}^{\infty} g_i\Delta u(t-i) \tag{2.1}$$

where $\Delta u(t)=u(t)-u(t-1)$ is the *control effort* at instant t. Therefore, it is natural to define the *predicted output* of the system at time $(t+k)$ calculated at instant t by

$$\hat{y}(t+k|t)=\sum_{i=1}^{\infty} g_i\Delta u(t+k-i)+\hat{n}(t+k|t)$$

where $\hat{n}(t+k|t)$ denotes the *predicted disturbance* at time $(t+k)$ calculated at instant t. We assume that the disturbance is constant along the prediction and it is equal to the difference of the measured output and the value given by the model (2.1), that is, $\hat{n}(t+k|t)=\hat{n}(t|t)=y_{\text{meas}}(t)-y_{\text{model}}(t)$. This implies that

$$\begin{aligned}&\hat{y}(t+k|t)\\&=\sum_{i=1}^{k} g_i\Delta u(t+k-i)+\sum_{i=k+1}^{\infty} g_i\Delta u(t+k-i)+y_{\text{meas}}(t)-\sum_{i=1}^{\infty} g_i\Delta u(t-i)\\&=\sum_{i=1}^{k} g_i\Delta u(t+k-i)+f(t+k)\end{aligned}$$

where

$$f(t+k)=y_{\text{meas}}(t)+\sum_{i=1}^{\infty}(g_{k+i}-g_i)\Delta u(t-i)$$

is the so-called *free response* of the system, i.e., the part of the response that does not depend on the future control actions. We assume that the system is stable, that is, $\lim_{t\to\infty} y(t)$ exists, so that there is N such that $g_i\approx g_N$ for $i\geqslant N$. We call N the *sampling time* (i.e., the coefficients of the step response tend to a constant value after N sampling periods). Clearly, if N exists, then the free response can be calculated as

$$f(t+k)=y(t)+\sum_{i=1}^{N}(g_{k+i}-g_i)\Delta u(t-i) \tag{2.2}$$

where we use the convention $g_i=g_N$ for $i\geqslant N$.

Suppose now that we want to predict the output of the system for p instants ahead, p is called the *prediction horizon*, and we take m control actions $(m\leqslant p)$ along this horizon. Then the predictions are

$$\hat{y}(t+1|t)=g_1\Delta u(t)+f(t+1)$$

$$\hat{y}(t+2|t) = g_2\Delta u(t) + g_1\Delta u(t+1) + f(t+2)$$
$$\vdots$$
$$\hat{y}(t+p|t) = \sum_{i=p-m+1}^{p} g_i\Delta u(t+p-i) + f(t+p),$$

which can be written in the matrix form

$$\hat{\mathbf{y}} = G\Delta\mathbf{u} + \mathbf{f}$$

where we denote $\hat{\mathbf{y}} = (\hat{y}(t+1|t),\dots,\hat{y}(t+p|t))$, $\Delta\mathbf{u} = (\Delta u(t),\dots,\Delta u(t+m-1))$ and $\mathbf{f} = (f(t+1),\dots,f(t+p))$, further,

$$G = \begin{bmatrix} g_1 & 0 & \dots & 0 \\ g_2 & g_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_m & g_{m-1} & \cdots & g_1 \\ \vdots & \vdots & \ddots & \vdots \\ g_p & g_{p-1} & \cdots & g_{p-m+1} \end{bmatrix}$$

is the *dynamic matrix* of the system (which is a $p \times m$ matrix).

Assume that a *reference trajectory* $w(t)$ is given and the control signal is determined such that the following (quadratic) *objective function* is minimized:

$$\begin{aligned} J &= \sum_{i=1}^{p}(\hat{y}(t+i|t) - w(t+i))^2 + \sum_{i=1}^{m}\lambda(\Delta u(t+i-1))^2 \\ &= |G\Delta\mathbf{u} + \mathbf{f} - \mathbf{w}|^2 + \lambda|\Delta\mathbf{u}|^2 \end{aligned} \tag{2.3}$$

with $\lambda \geqslant 0$. We note that the second sum of (2.3) penalizes the large control efforts. If there are no other constraints on the control variable then the minimum point of the function (2.3) is obtained by simple differentiation, and we find that

$$\Delta\mathbf{u} = (G^T G + \lambda I)^{-1} G^T(\mathbf{w} - \mathbf{f}). \tag{2.4}$$

(If there are other constraints on the control, for instance, $u_{\min} \leqslant u(t) \leqslant u_{\max}$ for every t, then the minimum may be obtained by numerical optimization.) Remember that only $u(t)$ is applied to the system, the control law is recalculated at the next instant by using the above method. Below we consider the method in two special cases.

2.2. Constant reference trajectory with $\lambda = 0$

In what follows, we suppose that the reference trajectory is constant, i.e., $w(t) = w \in \mathbb{C}$ is the *setpoint*. Further, let $p = m$ (i.e., at each instant of the prediction horizon a control action is taken) and $\lambda = 0$ in the objective function (2.3). In this case G is a square matrix thus (2.4) implies that if G^{-1} exists (which is equivalent to $g_1 \neq 0$) then the control effort vector should be $\Delta\mathbf{u} = G^{-1}(\mathbf{w} - \mathbf{f})$. Recall that we only need the first entry of $\Delta\mathbf{u}$. Observe that G^{-1} is a lower triangular matrix with upper left corner entry $1/g_1$ thus

$$u(t) - u(t-1) = \frac{1}{g_1}(w - f(t+1))$$

hence

$$u(t) = u(t-1) + \frac{1}{g_1}(w - f(t+1)).$$

Now from (2.2) we obtain that

$$u(t) = \frac{1}{g_1}(w - y(t)) + \sum_{i=1}^{N} \frac{2g_i - g_{i-1} - g_{i+1}}{g_1} u(t-i). \tag{2.5}$$

Recall that we use the convention $g_{N+1} = g_N$, further, $g_1 \neq 0$ is assumed.

2.3. Constant reference trajectory with $\lambda > 0$

Let us consider another special case when $\lambda > 0$, further, $p = m = 1$ (i.e., the prediction horizon is 1 and 1 control action is taken). Then $G = [g_1]$ so (2.4) implies that

$$\begin{aligned} u(t) - u(t-1) &= (G^T G + \lambda I)^{-1} G^T (w - f(t+1)) \\ &= \frac{g_1}{g_1^2 + \lambda}(w - f(t+1)). \end{aligned} \tag{2.6}$$

Assuming $g_1 \neq 0$ and introducing $0 < \mu = g_1^2/(g_1^2 + \lambda) \leqslant 1$, equation (2.6) may be written as

$$u(t) = u(t-1) + \frac{\mu}{g_1}(w - f(t+1))$$

and so

$$\begin{aligned} u(t) = {} & \frac{\mu}{g_1}(w - y(t)) + \left(\mu \frac{g_1 - g_2}{g_1} + 1\right) u(t-1) \\ & + \sum_{i=2}^{N} \mu \frac{2g_i - g_{i-1} - g_{i+1}}{g_1} u(t-i). \end{aligned} \tag{2.7}$$

Notice that for $\mu = 1$ (i.e., for $\lambda = 0$) equation (2.7) yields (2.5).

3. State-space representation and asymptotic output controllability

3.1. The continuous process

Suppose that a SISO linear continuous-time time-invariant system is given in the following state-sapce representation:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{3.1}$$

$$y(t) = Cx(t) \tag{3.2}$$

where $A \in \mathbb{C}^{n\times n}$ is the *state matrix*, $B \in \mathbb{C}^{n\times 1}$ is the *input matrix*, $C \in \mathbb{C}^{1\times n}$ is the *output matrix*, further, $x(t) \in \mathbb{C}^n$ is the *state variable*, $u(t) \in \mathbb{C}$ is the *control (input) variable* and $y(t) \in \mathbb{C}$ is the *observed (output) variable*, see [17]. We assume that A is diagonalizable and has no zero eigenvalues, i.e., there exists an invertible $P \in \mathbb{C}^{n\times n}$ such that $P^{-1}AP = \Lambda$ where Λ is diagonal with nonzero diagonal entries. Then the system (3.1)-(3.2) can be reformulated as

$$(P^{-1}x)^{\cdot} = (P^{-1}AP)(P^{-1}x) + (P^{-1}B)u \tag{3.3}$$

$$y = (CP)(P^{-1}x). \tag{3.4}$$

Therefore, by introducing the new state variable $P^{-1}x$, the new state matrix $P^{-1}AP = \Lambda$, the new input matrix $P^{-1}B$ and the new ouput matrix CP, we may assume that A is diagonal. If the control variable is constant, $u(t) = u \in \mathbb{C}$, then the solution of (3.1) is given by

$$x(t) = e^{At}x(0) + (e^{At} - I)A^{-1}Bu. \tag{3.5}$$

3.2. The sampled process

Now we construct a sampled version of the continuous process by measuring y at discrete instants (without noises). We choose a time step τ and in what follows, for simplicity, time is measured in integers, $t = j \in \mathbb{Z}$ means that we are at instant $j\tau$. Supposing that u is held constant along the interval $[j\tau, (j+1)\tau]$, the solution formula (3.5) yields the following discretized form of (3.1):

$$x(t) = e^{A\tau}x(t-1) + (e^{A\tau} - I)A^{-1}Bu(t-1). \tag{3.6}$$

The step response of the system is obtained by applying a step in the control variable, from (3.5) it follows that for $i = 0, 1, \dots, N$,

$$g_i = Cx(i) = Cx(0) + C((e^{A\tau})^i - I)A^{-1}B = C((e^{A\tau})^i - I)A^{-1}B \tag{3.7}$$

where $Cx(0) = y(0) = 0$ by assumption in case of generating the step response. We further assume that $g_i = g_N$ for $i > N$.

3.3. The DMC algorithm for the sampled process

We are interested in the case when the system (3.1) is stable, i.e., the eigenvalues of A lie in the open left half plane, so that at the stationary state $x = -A^{-1}Bu$. Therefore, if the output of system is driven into the setpoint $y = w$, then $w = -CA^{-1}Bu$ hence $u = -w/CA^{-1}B$ and $x = A^{-1}Bw/CA^{-1}B$. Thus, in case $CA^{-1}B \neq 0$, it is convenient to reformulate the recurrences (3.6) and (2.7) in terms of

$$\tilde{x}(t) = x(t) - \frac{A^{-1}Bw}{CA^{-1}B} \qquad \text{and} \qquad \tilde{u}(t) = u(t) + \frac{w}{CA^{-1}B}. \tag{3.8}$$

We obtain

$$\tilde{x}(t) = e^{A\tau}\tilde{x}(t-1) + (e^{A\tau} - I)A^{-1}B\tilde{u}(t-1), \tag{3.9}$$

$$\begin{aligned}\tilde{u}(t) = &-\frac{\mu}{g_1}C\tilde{x}(t) + \left(\mu\frac{g_1-g_2}{g_1}+1\right)\tilde{u}(t-1)\\ &+\sum_{i=2}^{N}\mu\frac{2g_i-g_{i-1}-g_{i+1}}{g_1}\tilde{u}(t-i).\end{aligned} \tag{3.10}$$

Finally, the substitution of equation (3.9) into (3.10) and $g_1 = C(e^{A\tau}-I)A^{-1}B$ yield the following recurrences

$$\tilde{x}(t) = e^{A\tau}\tilde{x}(t-1) + (e^{A\tau}-I)A^{-1}B\tilde{u}(t-1) \tag{3.11}$$

$$\begin{aligned}\tilde{u}(t) = &-\frac{\mu}{g_1}Ce^{A\tau}\tilde{x}(t-1) + \left(\mu\frac{g_1-g_2}{g_1}+1-\mu\right)\tilde{u}(t-1)\\ &+\sum_{i=2}^{N}\mu\frac{2g_i-g_{i-1}-g_{i+1}}{g_1}\tilde{u}(t-i).\end{aligned} \tag{3.12}$$

Introducing

$$\mathbf{v}(t) = (\tilde{x}(t), \tilde{u}(t), \ldots, \tilde{u}(t-N+1)) \in \mathbb{C}^{n+N} \tag{3.13}$$

the recurrences (3.11) and (3.12) may be written in the form

$$\mathbf{v}(t) = M_N\mathbf{v}(t-1) \tag{3.14}$$

where the $(N+n)\times(N+n)$ matrix M_N has the block-matrix form

$$M_N = \begin{bmatrix} e^{A\tau} & (e^{A\tau}-I)A^{-1}B & 0 & \ldots & 0 & 0\\ -\frac{\mu Ce^{A\tau}}{g_1} & \mu\frac{g_1-g_2}{g_1}+1-\mu & \mu\frac{2g_2-g_1-g_3}{g_1} & \ldots & \mu\frac{2g_{N-1}-g_{N-2}-g_N}{g_1} & \mu\frac{2g_N-g_{N-1}-g_{N+1}}{g_1}\\ 0 & 1 & 0 & \ldots & 0 & 0\\ 0 & 0 & 1 & \ldots & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \ldots & 1 & 0 \end{bmatrix}. \tag{3.15}$$

So we obtained that the Dynamic Matrix Control algorithm is equivalent to the iteration (or difference equation) (3.14).

3.4. Asymptotic output controllability

The iteration is said to be *(exponentially) stable* if $\mathbf{v}(t) \to \mathbf{0}$ as $t \to \infty$ for every initial value $\mathbf{v}(0)$. Regarding the stability of difference equations we recall the following well-known result, see [8].

THEOREM 1. *Let $Q \in \mathbb{C}^{k\times k}$. Then $\lim_{j\to\infty} Q^j z = 0$ for every $z \in \mathbb{C}^k$ if and only if $\lim_{j\to\infty} Q^j = 0$ and this occurs if and only if $\rho(Q) < 1$ where $\rho(Q) = \max_{i=1,\ldots,n}|\lambda_i|$ is the spectral radius of Q (sometimes such a Q is called a* discrete-time Hurwitz matrix*). Moreover, for every fixed norm $\|\cdot\|$ and for every $\varepsilon > 0$ there exists j_0 such that for $j \geqslant j_0$ and $z \in \mathbb{C}^k$*

$$\|Q^j z\| \leqslant (\rho(Q)+\varepsilon)^j\|z\|.$$

If the difference equation is exponentially stable, then it means that the DMC algorithm is also convergent, that is, for every setpoint w, it drives the output of the system into w as $t \to \infty$. We say that the system (3.1)-(3.2) is *asymptotically output controllable* with DMC (see [17]). Moreover, for fixed instant t,

$$\|\mathbf{v}(t+j)\| = \|M_N^j \mathbf{v}(t)\| \leqslant \|M_N^j\| \|\mathbf{v}(t)\|$$

and

$$|\tilde{y}(t+j)| = |C\tilde{x}(t+j)| \leqslant \|C\| \|\mathbf{v}(t+j)\|$$

hence

$$|y(t+j) - w| \leqslant \mathscr{C} \cdot (\rho(M_N) + \varepsilon)^j \quad (j \geqslant j_0)$$

where the constant $\mathscr{C}$ does not depend on ε and j_0 (it depends on C and $\mathbf{v}(t)$, that is, the state of the system at instant t and the controls applied before t).

We note that if the output can be driven to any setpoint in finite time by a suitable control then the system is called *output controllable*. A necessary and sufficient condition, the so-called Kalman rank condition, for output controllability of the system (3.1)-(3.2) is that the matrix

$$\left[CB \; CAB \; \ldots \; CA^{n-1}B\right]$$

has full rank (which is 1 in our case), see [17].

In Section 4, 5 and 6 we shall analyze the asymptotic output controllability for dimension $n = 1$ and $n = 2$, respectively. We compute the characteristic polynomial of the matrix M_N and study the location of its zeros. The locus of roots of a polynomial with respect to the unit circle centered at the origin has an extensive theory, several criteria are known, see [10] for a detailed discussion. However, our consideration will be self-contained and elementary. We give sufficient conditions on the convergence of the above control strategy and some estimates on the rate of convergence will be also established (see Theorem 2, 3 and 4).

4. The one-dimensional case with $\lambda = 0$

In this section we consider the case when $n = 1$ in the state space representation (3.1)-(3.2) so A, B, C are constants, further, $p = m$ (i.e., at each instant of the prediction horizon a control action is taken) and $\lambda = 0$ (so $\mu = 1$) in the objective function (2.3) (see Subsection 2.2). Furthermore, the cases $B = 0$ and $C = 0$ are out of interest since then the system is not controllable. The matrix M_N defined by (3.15) reduces to

$$\tilde{M}_N = \begin{bmatrix} e^{A\tau} & \frac{g_1}{C} & 0 & \ldots & 0 & 0 \\ -\frac{Ce^{A\tau}}{g_1} & \frac{g_1-g_2}{g_1} & \frac{2g_2-g_1-g_3}{g_1} & \ldots & \frac{2g_{N-1}-g_{N-2}-g_N}{g_1} & \frac{2g_N-g_{N-1}-g_{N+1}}{g_1} \\ 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 1 & 0 \end{bmatrix}. \tag{4.1}$$

Note that $\tilde{M}_N$ is not defined if $g_1 = 0$ or $C = 0$. Clearly, $g_1 = 0$ implies $B = 0$ or $C = 0$ by the unique solvability of the equation (3.1) which has been excluded above.

In what follows, we establish explicit bounds for the spectral radius of $\tilde{M}_N$ by calculating its characteristic polynomial and estimating its roots. For simplicity, denote $\alpha := e^{A\tau}$.

PROPOSITION 1. *The characteristic polynomial of the* $(N+1)\times(N+1)$ *matrix* $\tilde{M}_N$ *given by (4.1) is*

$$\tilde{p}_N(x) = (-1)^{N+1}(x^{N+1} - \alpha^N x + \alpha^N). \tag{4.2}$$

We recall a well-known result from linear algebra, for the proof see [8].

LEMMA 1. *The characteristic polynomial of the* $k\times k$ *matrix*

$$\begin{bmatrix} c_{k-1} & c_{k-2} & \dots & c_1 & c_0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}$$

is

$$(-1)^k(x^k - c_{k-1}x^{k-1} - \ldots - c_1 x - c_0).$$

Proof. [Proof of Proposition 1] By expanding $\det(\tilde{M}_N - xI)$ along the first column and using Lemma 1 it follows that

$$\begin{aligned}\tilde{p}_N(x) = (-1)^N(\alpha - x)\left(x^N - \frac{g_1 - g_2}{g_1}x^{N-1} - \sum_{i=2}^{N}\frac{2g_i - g_{i-1} - g_{i+1}}{g_1}x^{N-i}\right) \\ + (-1)^{N-1}\alpha x^{N-1}.\end{aligned} \tag{4.3}$$

Now (3.7) implies that

$$\frac{g_1 - g_2}{g_1} = \frac{\alpha - \alpha^2}{\alpha - 1} = -\alpha, \qquad \frac{g_N - g_{N-1}}{g_1} = \frac{\alpha^N - \alpha^{N-1}}{\alpha - 1} = \alpha^{N-1}, \tag{4.4}$$

further, for $i = 2, \dots, N-1$,

$$\frac{2g_i - g_{i-1} - g_{i+1}}{g_1} = \frac{2\alpha^i - \alpha^{i-1} - \alpha^{i+1}}{\alpha - 1} = \alpha^{i-1} - \alpha^i, \tag{4.5}$$

so that from (4.3) it follows

$$\begin{aligned}\tilde{p}_N(x) &= (-1)^N(\alpha - x)\left(x^N + \alpha x^{N-1} + \sum_{i=2}^{N-1}(\alpha^i - \alpha^{i-1})x^{N-i} - \alpha^{N-1}\right) \\ &\quad + (-1)^{N-1}\alpha x^{N-1} \\ &= (-1)^{N+1}(x^{N+1} - \alpha^N x + \alpha^N).\end{aligned}$$

REMARK 1. Observe that $\tilde{p}_N$ does not depend on C which is not surprising, since in the beginning one may suppose $C = 1$, otherwise we take $x(t)/C$ instead of $x(t)$ in (3.1)-(3.2). Moreover, in the computation of p_N we supposed $A \neq 0$, however, it is easily seen that the formula for p_N is also valid in this case (by passing to the limit).

Furthermore, we see that if $|\alpha| < 1$ (i.e., $\operatorname{Re} A < 0$) and N is sufficiently large, then p_N is "close" to the polynomial $(-1)^{N+1}x^{N+1}$ which has only zero roots. Therefore, it is plausible to think that for large N the roots of $\tilde{p}_N$ are "close" to 0, hence the roots lie inside the unit circle. This argument can be made precise by using the well-known Rouché's theorem, see [1].

LEMMA 2. (Rouché's theorem) *Let Ω be a bounded region of the complex plane and $\gamma \subset \Omega$ be a Jordan-curve (simple closed curve). Suppose that $f_0, f_1 : \Omega \to \mathbb{C}$ are holomorphic functions such that $|f_0(z) - f_1(z)| < |f_0(z)|$ for $z \in \gamma$. Then f_0 and f_1 have the same number of zeros (counted by multiplicity) inside γ.*

PROPOSITION 2. *Suppose $|\alpha| < 1$. Then the absolute value of the (real or complex) roots of $\tilde{p}_N$ given by* (4.2) *are less than*

$$R = 3^{\frac{1}{N+1}} |\alpha|^{\frac{N}{N+1}}. \tag{4.6}$$

Proof. We apply Rouché's theorem with $f_0(z) = (-1)^{N+1} z^{N+1}$, $f_1(z) = p_N(z)$ and $\gamma = \{|z| = R\}$. Since for $z \in \gamma$ we have $|z| < 2$ thus

$$|f_0(z) - f_1(z)| = |\alpha|^N |z - 1| < 3|\alpha|^N = |z|^{N+1} = |f_0(z)|,$$

therefore, $\tilde{p}_N(z)$ has the same number of roots inside γ as z^{N+1}, i.e., all the $(N+1)$ roots of $\tilde{p}_N$ lie inside γ.

Since for $|\alpha| < 1$ and sufficiently large N the bound (4.6) is less than 1, in view of the arguments of Subsection 3.4 we obtain the main result of this section.

THEOREM 2. *Consider the system described by* (3.1)-(3.2) *in one dimension with A, B, C nonzero constants. Suppose that we want to drive the output of the system into the setpoint w by using the Dynamic Matrix Control strategy with $p = m$ (i.e., at each instant of the prediction horizon a control action is taken) and $\lambda = 0$ in the objective function* (2.3)*: we measure y (without noises) at discrete instants with constant time step $\tau > 0$ and the stepwise constant control u is determined by* (2.5) *where the g_i constants are obtained from the step response of the system. If the system* (3.1) *is stable, i.e.* $\operatorname{Re} A < 0$, *then there exists a sufficiently large sampling time N such that the method drives the output of the system into the desired setpoint as $t \to \infty$, i.e., the system is asymptotically output controllable with DMC. Moreover, for every fixed t and $\varepsilon > 0$ small enough there exists k_0 such that the following estimate holds:*

$$|y(t+k) - w| \leqslant \mathscr{C} \cdot (R_1 + \varepsilon)^k \quad (k \geqslant k_0)$$

where

$$R_1 = 3^{\frac{1}{N+1}} (e^{A\tau})^{\frac{N}{N+1}}$$

and the constant $\mathscr{C}$ does not depend on ε and k_0 (it depends on C, on the state of the system at instant t and on the controls applied before t).

REMARK 2. As it follows from the arguments of Subsection 3.4, the system (3.1)-(3.2) is output controllable in one dimension if the matrix $[CB]$ has full rank, i.e., $CB \neq 0$. So Theorem 2 briefly says that a one-dimensional stable system which is output controllable is also asymptotically output controllable with the DMC strategy.

5. The one-dimensional case with $\lambda > 0$

We suppose that in the objective function $\lambda > 0$, further, $p = m = 1$, i.e., the prediction horizon is 1 and 1 control action is taken (see Subsection 2.3). Then the method reduces to the iteration (3.14) with the matrix (3.15):

$$M_N = \begin{bmatrix} \alpha & \frac{g_1}{C} & 0 & \dots & 0 & 0 \\ -\frac{\mu C\alpha}{g_1} & \mu\frac{g_1-g_2}{g_1}+1-\mu & \mu\frac{2g_2-g_1-g_3}{g_1} & \dots & \mu\frac{2g_{N-1}-g_{N-2}-g_N}{g_1} & \mu\frac{2g_N-g_{N-1}-g_{N+1}}{g_1} \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix} \tag{5.1}$$

where $\alpha = e^{A\tau}$ as in the previous section. We calculate the characteristic polynomial of $\tilde{M}_N$ and show that if μ is sufficiently close to 1 (i.e., λ is close to 0) then the eigenvalues of $\tilde{M}_N$ lie inside the unit circle.

PROPOSITION 3. *Let $N \geqslant 2$. Then the characteristic polynomial of the $(N+1)\times(N+1)$ matrix $\tilde{M}_N$ defined by (5.1) is*

$$\begin{aligned} p_N(x) = (-1)^{N+1}\big(&x^{N+1} - (1-\mu)(\alpha+1)x^N \\ &+ (1-\mu)\alpha x^{N-1} - \mu\alpha^N x + \mu\alpha^N\big). \end{aligned} \tag{5.2}$$

Proof. Analogously to the proof of Proposition 1, by expanding the determinant $\det(M_N - xI)$ along the first column and using Lemma 1 and the expressions (4.4), (4.5) it follows that

$$\begin{aligned} p_N(x) &= (-1)^N(\alpha - x)\Bigg(x^N + (\mu\alpha - 1 + \mu)x^{N-1} \\ &\qquad + \sum_{i=2}^{N-1}\mu(\alpha^i - \alpha^{i-1})x^{N-i} - \mu\alpha^{N-1}\Bigg) + (-1)^{N-1}\mu\alpha x^{N-1} \\ &= (-1)^{N+1}(x^{N+1} - (1-\mu)(\alpha+1)x^N + (1-\mu)\alpha x^{N-1} - \mu\alpha^N x + \mu\alpha^N). \end{aligned}$$

REMARK 3. For $\mu = 1$ (i.e., for $\lambda = 0$), we have $p_N = \tilde{p}_N$, and for $\mu = 0$ (i.e., as $\lambda \to +\infty$), $p_N(x) = (-1)^{N+1}x^{N-1}(x-\alpha)(x-1)$ which has all roots on the closed unit disk for $|\alpha| < 1$. Therefore, we expect that in case $|\alpha| < 1$ and for sufficiently large N, the roots of p_N lie inside the unit circle.

PROPOSITION 4. *For every $|\alpha| < 1$ there exist a sufficiently large N and $0 \leqslant \mu^* < 1$ such that for all $\mu^* \leqslant \mu \leqslant 1$ the roots of the polynomial p_N lie inside the circle $\{|z| = R\}$ where R is given by (4.6).*

Proof. By Proposition 2 the roots of $\tilde{p}_N$ lie inside a circle centered at the origin having radius R given by (4.6). If $|\alpha| < 1$ and N is sufficiently large, this radius is less than 1. We show that if $\mu \geqslant \mu^*$ for some μ^*, then the roots of p_N lie inside this circle, as well. We apply Rouché's theorem with $f_0(z) = \tilde{p}_N(z)$, $f_1(z) = p_N(z)$ and $\gamma = \{|z| = R\}$. We have to verify that for $|z| = R$,

$$(1-\mu)|(\alpha+1)z^N - \alpha z^{N-1} - \alpha^N z + \alpha^N| < |z^{N+1} - \alpha^N z + \alpha^N|. \tag{5.3}$$

For $|\alpha| < 1$ and $|z| = R < 1$ it follows that

$$|z^{N+1} - \alpha^N z + \alpha^N| \geqslant |z|^{N+1} - |\alpha|^N - |\alpha|^N = |\alpha|^N > 0,$$

further, obviously

$$(1-\mu)|(\alpha+1)z^N - \alpha z^{N-1} - \alpha^N z + \alpha^N| \leqslant (1-\mu)5R^N,$$

therefore, (5.3) holds true if

$$(1-\mu)5R^N < |\alpha|^N,$$

or equivalently

$$1-\mu < \frac{1}{5}3^{-\frac{N}{N+1}}|\alpha|^{\frac{1}{N+1}}.$$

Now, the main result of this section follows.

THEOREM 3. *Consider the system described by (3.1)-(3.2) in one dimension with nonzero constants A, B, C. Suppose that we want to drive the output of the system into the setpoint w by using the Dynamic Matrix Control strategy with $p = m = 1$ (i.e., the prediction horizion is 1 and 1 control action is taken) and $\lambda > 0$ in the objective function (2.3): we measure y (without noises) at discrete instants with constant time step $\tau > 0$ and the stepwise constant control u is determined by (2.7) where the g_i constants are obtained from the step response of the system. If the system (3.1) is stable, i.e.* $\mathrm{Re}\,A < 0$, *then there exist a sufficiently large sampling time N and $\lambda^* > 0$ such that for every $0 \leqslant \lambda \leqslant \lambda^*$ the method drives the output of the system into the desired setpoint as $t \to \infty$, i.e., the system is asymptotically output controllable with DMC. Moreover, for every fixed t and $\varepsilon > 0$ small enough there exists k_0 such that the following estimate holds:*

$$|y(t+k) - w| \leqslant \mathscr{C} \cdot (R_1 + \varepsilon)^k \quad (k \geqslant k_0)$$

where

$$R_1 = 3^{\frac{1}{N+1}} (e^{A\tau})^{\frac{N}{N+1}}$$

and the constant $\mathscr{C}$ *does not depend on* ε *and* k_0 *(it depends on* C*, on the state of the system at instant* t *and on the controls applied before* t*).*

6. Two-dimensional case

In this Section we suppose that $n = 2$ in the representation (3.1)-(3.2) of the process, $p = m$ (i.e., at each instant of the prediction horizon a control action is taken) and $\lambda = 0$ in the objective function (2.3). Let

$$A = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad C = \begin{bmatrix} c_1 & c_2 \end{bmatrix}.$$

(Recall that $CA^{-1}B \neq 0$ is supposed, in order to $\tilde{x}(t)$ and $\tilde{u}(t)$ be well-defined, see (3.8).) Then by (3.7), for $i = 0, 1, \ldots, N$ we have

$$g_i = C((e^{A\tau})^i - I)A^{-1}B = c_1 b_1 \frac{\alpha_1^i - 1}{\lambda_1} + c_2 b_2 \frac{\alpha_2^i - 1}{\lambda_2} \tag{6.1}$$

where we used the notation $\alpha_j = e^{\lambda_j \tau}$ $(j = 1, 2)$. As before, we assume that $g_i = g_N$ for $i > N$. This case the matrix M_N defined by (3.15) takes the form

$$M_N = \begin{bmatrix} \alpha_1 & 0 & b_1 \frac{\alpha_1 - 1}{\lambda_1} & 0 & \ldots & 0 & 0 \\ 0 & \alpha_2 & b_2 \frac{\alpha_2 - 1}{\lambda_2} & 0 & \ldots & 0 & 0 \\ -\frac{c_1 \alpha_1}{g_1} & -\frac{c_2 \alpha_2}{g_1} & \frac{g_1 - g_2}{g_1} & \frac{2g_2 - g_1 - g_3}{g_1} & \ldots & \frac{2g_{N-1} - g_{N-2} - g_N}{g_1} & \frac{2g_N - g_{N-1} - g_{N+1}}{g_1} \\ 0 & 0 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & 1 & 0 \end{bmatrix}. \tag{6.2}$$

An analogue of Proposition 1 is the following.

PROPOSITION 5. *Let* $N \geqslant 4$*. Then the characteristic polynomial of the* $(N+2) \times (N+2)$ *matrix* M_N *given by (6.2) is*

$$p_N(x) = (-1)^{N+2} \left(x^{N+2} - \left(\alpha_1 + \alpha_2 + \frac{g_1 - g_2}{g_1} \right) x^{N+1} + \gamma_2 x^2 + \gamma_1 x + \gamma_0 \right)$$

where $\gamma_i = \mathscr{O}(|\alpha_1|^N + |\alpha_2|^N)$.

Proof. By expanding $\det(M_N - xI)$ along the first two columns we find that

$$\begin{aligned}
p_N(x) &= (\alpha_1 - x)(\alpha_2 - x)q_N(x) + (\alpha_1 - x)\frac{c_2 b_2 \alpha_2(\alpha_2 - 1)}{g_1 \lambda_2}(-x)^{N-1} \\
&\quad + (\alpha_2 - x)\frac{c_1 b_1 \alpha_1(\alpha_1 - 1)}{g_1 \lambda_1}(-x)^{N-1} \\
&= (\alpha_1 - x)(\alpha_2 - x)q_N(x) + (-1)^{N+1}\frac{g_1 - g_2}{g_1}x^N + (-1)^{N+1}\alpha_1\alpha_2 x^{N-1}
\end{aligned}$$

where $q_N(x)$ is the characteristic polynomial of the $N \times N$ matrix

$$\begin{bmatrix}
\frac{g_1-g_2}{g_1} & \frac{2g_2-g_1-g_3}{g_1} & \dots & \frac{2g_{N-1}-g_{N-2}-g_N}{g_1} & \frac{2g_N-g_{N-1}-g_{N+1}}{g_1} \\
1 & 0 & \dots & 0 & 0 \\
0 & 1 & \dots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \dots & 1 & 0
\end{bmatrix}$$

which is

$$\begin{aligned}
q_N(x) = (-1)^N \Bigg(x^N - \frac{g_1 - g_2}{g_1}x^{N-1} - \frac{2g_2 - g_1 - g_3}{g_1}x^{N-2} - \dots \\
- \frac{2g_N - g_{N-1} - g_{N+1}}{g_1} \Bigg)
\end{aligned}$$

due to Lemma 1. Clearly, the constant term in p_N is

$$\gamma_0 = (-1)^{N+1}\alpha_1\alpha_2\frac{g_N - g_{N-1}}{g_1},$$

the coefficient of x is

$$\gamma_1 = (-1)^{N+2}(\alpha_1 + \alpha_2)\frac{g_N - g_{N-1}}{g_1} + (-1)^{N+1}\alpha_1\alpha_2\frac{2g_{N-1} - g_{N-2} - g_N}{g_1},$$

and the coefficient of x^2 is

$$\begin{aligned}
\gamma_2 = (-1)^{N+1}\frac{g_N - g_{N-1}}{g_1} + (-1)^{N+2}(\alpha_1 + \alpha_2)\frac{2g_{N-1} - g_{N-2} - g_N}{g_1} \\
+ (-1)^{N+1}\alpha_1\alpha_2\frac{2g_{N-2} - g_{N-3} - g_{N-1}}{g_1}.
\end{aligned}$$

Now by (6.1) it is clear that $|\gamma_i| \leqslant \beta \cdot (|\alpha_1|^N + |\alpha_2|^N)$ where the constant β may depend on c_i, b_i, α_i $(i = 1,2)$ but not on N. The coefficient of x^{N+1} is also clear since it is $-\mathrm{Tr} M_N$. So it remains to verify that the coefficient of x^j is zero for j not being the above specific values. Then the coefficient of x^j is

$$\gamma_j = (-1)^{N+1}\frac{2g_{N-j+2} - g_{N-j} - g_{N-j+3}}{g_1}$$

$$+(-1)^{N+2}(\alpha_1+\alpha_2)\frac{2g_{N-j+1}-g_{N-j}-g_{N-j+2}}{g_1}$$
$$+(-1)^{N+1}\alpha_1\alpha_2\frac{2g_{N-j}-g_{N-j-1}-g_{N-j+1}}{g_1}.$$

Since in general $g_i - g_{i-1} = C((e^{A\tau})^i - (e^{A\tau})^{i-1})A^{-1}B$, and the characteristic polynomial of $e^{A\tau}$ is $x^2-(\alpha_1+\alpha_2)x+\alpha_1\alpha_2$, therefore, by the Cayley-Hamilton theorem it follows

$$\begin{aligned}(g_i-g_{i-1})&-(\alpha_1+\alpha_2)(g_{i-1}-g_{i-2})+\alpha_1\alpha_2(g_{i-2}-g_{i-3})\\ &=C\Big((e^{A\tau})^{i-2}-(e^{A\tau})^{i-3}\Big)\Big((e^{A\tau})^2-(\alpha_1+\alpha_2)e^{A\tau}+\alpha_1\alpha_2\Big)A^{-1}B\\ &=0\end{aligned}$$

which yields $\gamma_j = 0$.

PROPOSITION 6. *Suppose $|\alpha_i|<1$ $(i=1,2)$, further,*

$$\left|\alpha_1+\alpha_2+\frac{g_1-g_2}{g_1}\right|<1.$$

Then for every $\varepsilon>0$ there is a sufficiently large N^* such that for all $N \geqslant N^*$ all the $(N+2)$ eigenvalues of the matrix (6.2) lie inside the circle centered at the origin having radius $R_2+\varepsilon$ where

$$R_2=\left|\alpha_1+\alpha_2+\frac{g_1-g_2}{g_1}\right|.$$

Proof. We apply Rouché's theorem for $p_N(z) =: f_1(z)$ the characteristic polynomial of M_N and

$$f_0(z) := (-1)^{N+2}\left(z^{N+2}-\left(\alpha_1+\alpha_2+\frac{g_1-g_2}{g_1}\right)z^{N+1}\right)$$

which has all zeros inside the circle with radius $R_2+\varepsilon$. Since for fixed $|z|$, by Proposition 5 it follows

$$|f_0(z)-f_1(z)| = \mathscr{O}(|\alpha_1|^N+|\alpha_2|^N)\to 0 \quad \text{as} \quad N\to\infty,$$

therefore, for sufficiently large N and $|z| = R_2+\varepsilon$,

$$|f_0(z)-f_1(z)| < |f_0(z)| = \left|z-(\alpha_1+\alpha_2)-\frac{g_1-g_2}{g_1}\right| \geqslant \varepsilon.$$

Hence f_0 and f_1 have the same number of zeros inside the circle $\{|z| = R_2+\varepsilon\}$.

Now we obtain the main result of this section.

THEOREM 4. *Consider the system described by* (3.1)-(3.2) *in two dimensions with diagonal matrix A having eigenvalues λ_1, λ_2. Suppose that we want to drive the output of system into the setpoint w by using the Dynamic Matrix Control strategy with $p = m$ (at each instant of the prediction horizon a control action is taken) and $\lambda = 0$ in the objective function* (2.3)*: we measure y (without noises) at discrete instants with constant time step $\tau > 0$ and the stepwise constant control u is determined by* (2.7) *where the g_i constants are obtained from the step response of the system and $g_1 \neq 0$. If the system* (3.1) *is stable, i.e. $\operatorname{Re}\lambda_i < 0$ $(i = 1,2)$, further, $CA^{-1}B \neq 0$ and*

$$R_2 := \left| e^{\lambda_1 \tau} + e^{\lambda_2 \tau} + \frac{g_1 - g_2}{g_1} \right| < 1, \tag{6.3}$$

then there exists a sufficiently large sampling time N such that the method drives the output of the system into the desired setpoint as $t \to \infty$, i.e., the system is asymptotically output controllable with DMC. Moreover, for every fixed t and $\varepsilon > 0$ small enough there exists k_0 such that the following estimate holds:

$$|y(t+k) - w| \leqslant \mathscr{C} \cdot (R_2 + \varepsilon)^k \quad (k \geqslant k_0)$$

where the constant $\mathscr{C}$ does not depend on ε and k_0 (it depends on C, on the state of the system at instant t and on the controls applied before t).

REMARK 4. If $\operatorname{Re}\lambda_i < 0$ $(i = 1,2)$ then

$$e^{\lambda_1 \tau} + e^{\lambda_2 \tau} + \frac{g_1 - g_2}{g_1} \to 0 \quad \text{as} \quad \tau \to \infty,$$

so that the condition (6.3) holds for every sufficiently large τ. Clearly, in case $\operatorname{Re}\lambda_i$ is close to 0, τ should be large. Intuitively, if the system stabilizes slowly then for short time step the step response model does not apply, we should let the system move into the state where the model already applies.

Observe that if $g_1 = 0$ and $CA^{-1}B = 0$ simultaneously for some $\tau > 0$, then by (6.1),

$$c_1 b_1/\lambda_1 = -c_2 b_2/\lambda_2 \quad \text{and} \quad c_1 b_1 e^{\lambda_1 \tau}/\lambda_1 = -c_2 b_2 e^{\lambda_2 \tau}/\lambda_2$$

thus it follows that $g_1 = 0$ for every $\tau > 0$. This means that the control u has no effect on the output so the system could not be controlled.

From the arguments of Subsection 3.4 it follows that the system (3.1)-(3.2) is not output controllable if and only if the matrix $\left[CB\ CAB\right]$, which is now a vector, is not of full rank, i.e., $CB = 0$ and $CAB = 0$. This implies that $g_1 = 0$ independently of the choice of $\tau > 0$.

Finally, we remark that the above analysis might be carried out for arbitrary dimension n. Indeed, it is not so difficult to see that in general the characteristic polynomial of the matrix (3.15) takes the form

$$\begin{aligned} p_N(x) = (-1)^{N+n}\big(x^{N+n} + \gamma_{N+n-1}x^{N+n-1} + \cdots + \gamma_{N+1}x^{N+1} \\ + \gamma_{n+1}x^{n+1} + \cdots + \gamma_1 x + \gamma_0\big) \end{aligned}$$

where $\gamma_i \to 0$ as $N \to \infty$ for $i = 1,\dots,n+1$. By using the results of [10] on the location of the zeros of a polynomial, one may obtain sufficient conditions for M_N being a discrete-time Hurwitz matrix. Algoritmically these conditions are easy to check, however, the formulas become more complicated, that is why we considered above only dimensions one and two.

7. Discussion and conclusions

In this paper we studied the so-called Dynamic Matrix Control (DMC) strategy which has many industrial applications. DMC is a kind of Model Predictive Control which is based on the step response model of the process. The main objective was to find sufficient conditions for asymptotic output controllability, that is, when the method drives the output of the system into the desired setpoint as time goes to infinity. We have shown that a process governed by a one-dimensional stable dynamical system is asymptotically output controllable with the method. Moreover, if the system is not stable, then it is not necessarily asymptotically output controllable with DMC. For two-dimensional systems sufficient condition for asymptotic output controllability has been given. In addition, estimates on the rate of convergence has also been established. Such an analysis of the DMC seems to be completely new.

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