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EXISTENCE OF POSITIVE SOLUTIONS FOR A QUASILINEAR ELLIPTIC SYSTEM OF p–KIRCHHOFF TYPE

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Abstract. In this paper, we consider the existence of positive solutions to the following p-Kirchhoff-type system

$$\begin{cases} -M\Big(\int_\Omega |\nabla u|^p dx\Big)\Delta_p u = g(x)|u|^{q-2}u + \frac{\alpha}{\alpha+\beta}|u|^{\alpha-2}u|v|^\beta, & x\in\Omega, \\ -M\Big(\int_\Omega |\nabla u|^p dx\Big)\Delta_p v = h(x)|v|^{q-2}v + \frac{\beta}{\alpha+\beta}|u|^{\alpha}|v|^{\beta-2}v, & x\in\Omega, \\ u=v=0, & x\in\partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$, $M(s)=a+bs^k$, $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator, $\alpha>1$, $\beta>1$, $1<p<q<\alpha+\beta<p^*=\frac{Np}{N-p}$.

1. Introduction

In this paper, we deal with the nonlocal elliptic system of the p-Kirchhoff type given by

$$\begin{cases} -M\Big(\int_\Omega |\nabla u|^p dx\Big)\Delta_p u = g(x)|u|^{q-2}u + \frac{\alpha}{\alpha+\beta}|u|^{\alpha-2}u|v|^\beta, & x\in\Omega, \\ -M\Big(\int_\Omega |\nabla u|^p dx\Big)\Delta_p v = h(x)|v|^{q-2}v + \frac{\beta}{\alpha+\beta}|u|^{\alpha}|v|^{\beta-2}v, & x\in\Omega, \\ u=v=0, & x\in\partial\Omega, \end{cases} \tag{1.1}$$

where $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator, $\alpha>1$, $\beta>1$ and $1<p<q<\alpha+\beta<p^*=\frac{Np}{N-p}$.

In recent years, there have been many papers concerned with the existence of positive solutions for Kirchhoff equation

$$\begin{cases} -M\Big(\int_\Omega |\nabla u|^2 dx\Big)\Delta u = f(x,u) & x\in\Omega, \\ u=0 & x\in\partial\Omega, \end{cases} \tag{1.2}$$

which is related to the stationary analogue of the Kirchhoff equation

$$u_{tt} - M\Big(\int_\Omega |\nabla_x u|^2 dx\Big)\Delta_x u = f(x,t)$$

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where $M(s)=a+bs,\ a>0,\ b>0$. It was proposed by Kirchhoff [2] as an extension of the classical D'Alembert's wave equation for free vibrations of elastic strings.

Some interesting studies of problem (1.2) by variational methods can be found in [1,3,6,7,9,10]. As for quasilinear problems, [3] studied the following equation

$$\begin{cases} -M\Big(\int_\Omega |\nabla u|^p dx\Big)\Delta_p u = f(x,u) & x\in\Omega,\\ u=0 & x\in\partial\Omega, \end{cases} \tag{1.3}$$

where $M(s)=a+bs^k,\ f(x,u)=\lambda h_1(x)|u|^{q-2}u+h_2(x)|u|^{r-2}u+h_3(x),\ 1<q<p<r<p^*,\ 0\leqslant k<\frac{p}{N-p},\ p(k+1)<r.$ The authors proved (1.3) has at least two nontrivial weak solutions when $||h_3||_\mu<m_0$.

In [4], the authors established the existence of a weak solution for the following system

$$\begin{cases} -[M_1(\int_\Omega |\nabla u|^p dx)]^{p-1}\Delta_p u = f(u,v)+\rho_1(x) & x\in\Omega,\\ -[M_2(\int_\Omega |\nabla v|^p dx)]^{p-1}\Delta_p v = g(u,v)+\rho_2(x) & x\in\Omega,\\ \frac{\partial u}{\partial \eta}=\frac{\partial v}{\partial \eta}=0 & x\in\partial\Omega, \end{cases} \tag{1.4}$$

where $M_1(t),M_2(t)\geqslant m_0>0$.

Motivated by the results of the above cited papers, we shall attempt to treat problem (1.1) and extend the results of the literature [4].

In this paper, we make the following assumptions:

$(A1)$ $M(s)=a+bs^k,\ a,b>0,\ k>0$;

$(A2)$ $1<p<q<\alpha+\beta<p^*,\ p(k+1)<q$;

$(A3)$ $h(x),H(x)\in L^{\delta+\gamma}(\Omega)$ are nonnegative with $\delta=\frac{p^*}{p^*-q}$ and γ is a small positive number.

Our main result is given as follows.

THEOREM 1. *Under assumptions $(A_1)-(A_3)$, problem (1.1) admits at least one positive solution $(u_0,v_0)\in W_0^{1,p}(\Omega)\times W_0^{1,p}(\Omega)$.*

2. Preliminaries

Let $X=W_0^{1,p}(\Omega)\times W_0^{1,p}(\Omega)$ be the Sobolev space endowed with the norm

$$||(u,v)||=\Big(\int_\Omega (|\nabla u|^p+|\nabla v|^p)dx\Big)^{\frac{1}{p}}$$

and $|u|_r$ denotes the norm in $L^r(\Omega)$, i.e.

$$|u|_r=\Big(\int_\Omega |u|^r dx\Big)^{\frac{1}{r}}$$

We shall look for solutions of (1.1) by finding critical points of the energy functional $I: X\to\mathbb{R}$ given by

$$I(u,v)=\frac{1}{p}\widehat{M}(\int_\Omega|\nabla u|^p dx)+\frac{1}{p}\widehat{M}(\int_\Omega|\nabla v|^p dx)\\ -\frac{1}{\alpha+\beta}\int_\Omega|u|^\alpha|v|^\beta dx-\frac{1}{q}\int_\Omega(g(x)|u|^q+h(x)|v|^q)dx,$$

where $\widehat{M}(t)=\int_0^t M(s)ds=at+\frac{b}{k+1}t^{k+1}$. It is well known that the functional $I(u,v)\in C^1(X,\mathbb{R})$. For any $(\varphi_1,\varphi_2)\in X$, there holds

$$\begin{aligned}&\langle I'(u,v),(\varphi_1,\varphi_2)\rangle\\ &\quad=M\Big(\int_\Omega|\nabla u|^p dx\Big)\int_\Omega|\nabla u|^{p-2}\nabla u\nabla\varphi_1 dx\\ &\qquad+M\Big(\int_\Omega|\nabla v|^p dx\Big)|\nabla v|^{p-2}\nabla v\nabla\varphi_2)dx\\ &\qquad-\int_\Omega(g(x)|u|^{q-2}u\varphi_1+h(x)|v|^{q-2}v\varphi_2)dx\\ &\qquad-\frac{\alpha}{\alpha+\beta}\int_\Omega|u|^{\alpha-2}u|v|^\beta\varphi_1 dx-\frac{\beta}{\alpha+\beta}\int_\Omega|u|^\alpha|v|^{\beta-2}v\varphi_2 dx.\end{aligned}$$

Consider the Nehari manifold

$$N=\{(u,v)\in X\backslash\{(0,0)\}|\langle I'(u,v),(u,v)\rangle=0\}.$$

Thus, $(u,v)\in N$ if and only if

$$M\Big(\int_\Omega|\nabla u|^p dx\Big)\int_\Omega|\nabla u|^p dx+M\Big(\int_\Omega|\nabla v|^p dx\Big)\int_\Omega|\nabla v|^p dx\\ -\int_\Omega(g(x)|u|^q+h(x)|v|^q)dx-\int_\Omega|u|^\alpha|v|^\beta dx=0.$$

Note that the Nehari manifold N contains all nontrivial weak solutions of (1.1).

Denote

$$S_{\alpha,\beta}=\inf_{u,v\in W_0^{1,p}(\Omega)\backslash\{0\}}\frac{||(u,v)||^p}{(\int_\Omega|u|^\alpha|v|^\beta dx)^{\frac{p}{\alpha+\beta}}}.$$

S is the best Sobolev constant defined by

$$S=\inf_{u\in W_0^{1,p}(\Omega)\backslash\{0\}}\frac{\int_\Omega|\nabla u|^p dx}{(\int_\Omega|u|^{p^*}dx)^{\frac{p}{p^*}}}>0.$$

Note that I is not bounded from below on X. But from the following lemma, we have that I is bounded from below on the Nehari manifold N.

LEMMA 1. *The energy functional I is bounded from below on N.*

Proof. For any $(u,v)\in N$, we have

$$I(u,v)=\frac{a}{p}||(u,v)||^p+\frac{b}{p(k+1)}[(\int_\Omega|\nabla u|^p dx)^{k+1}+(\int_\Omega|\nabla v|^p dx)^{k+1}]$$

$$
\begin{aligned}
&\quad - \frac{1}{\alpha+\beta}\int_\Omega |u|^\alpha |v|^\beta dx - \frac{a}{q}||(u,v)||^p - \frac{b}{q}[(\int_\Omega |\nabla u|^p dx)^{k+1} \\
&\quad + (\int_\Omega |\nabla v|^p dx)^{k+1}] + \frac{1}{q}\int_\Omega |u|^\alpha |v|^\beta dx \\
&= (\frac{a}{p} - \frac{a}{q})||(u,v)||^p + (\frac{b}{p(k+1)} - \frac{b}{q})\Big[(\int_\Omega |\nabla u|^p dx)^{k+1} \\
&\quad + (\int_\Omega |\nabla v|^p dx)^{k+1}\Big] + (\frac{1}{q} - \frac{1}{\alpha+\beta})\int_\Omega |u|^\alpha |v|^\beta dx > 0
\end{aligned}
$$

Thus, I is bounded from below on N. □

Then, we define

$$
\theta = \inf_{(u,v)\in N} I(u,v).
$$

LEMMA 2. *(i) There are* ρ *,* $r_0 > 0$ *such that* $I(u,v) \geqslant r_0$ *for* $||(u,v)|| = \rho$*.*
(ii) There exists $(\overline{u},\overline{v}) \in X\backslash\{(0,0)\}$ *such that* $||(\overline{u},\overline{v})|| > \rho$ *and* $I(\overline{u},\overline{v}) < 0$*.*

Proof. (i) By Hölder's inequality ($q_1 = \frac{p^*}{p^*-q}$, $q_2 = \frac{p^*}{q}$, $\frac{1}{q_1} + \frac{1}{q_2} = 1$) and the Sobolev embedding theorem, we have

$$
\begin{aligned}
I(u,v) &= \frac{a}{p}||(u,v)||^p + \frac{b}{p(k+1)}[(\int_\Omega |\nabla u|^p dx)^{k+1} + (\int_\Omega |\nabla v|^p dx)^{k+1}] \\
&\quad - \frac{1}{\alpha+\beta}\int_\Omega |u|^\alpha |v|^\beta dx - \frac{1}{q}\int_\Omega (g(x)|u|^q + h(x)|v|^q) dx \\
&\geqslant \frac{a}{p}||(u,v)||^p + \frac{b}{2^k p(k+1)}(\int_\Omega |\nabla u|^p dx + \int_\Omega |\nabla v|^p dx)^{k+1} \\
&\quad - \frac{1}{\alpha+\beta}\int_\Omega |u|^\alpha |v|^\beta dx - \frac{1}{q}\int_\Omega (g(x)|u|^q + h(x)|v|^q) dx \\
&\geqslant \frac{a}{p}||(u,v)||^p + \frac{b}{2^k p(k+1)}||(u,v)||^{p(k+1)} - \frac{1}{\alpha+\beta} S_{\alpha,\beta}^{-\frac{\alpha+\beta}{p}} ||(u,v)||^{\alpha+\beta} \\
&\quad - \frac{1}{q}\max\{||g||_\infty, ||h||_\infty\}|\Omega|^{\frac{p^*-q}{p^*}} S^{-\frac{q}{p}} ||(u,v)||^q.
\end{aligned}
$$

Since $p < p(k+1) < q < \alpha+\beta$, there are ρ, $r_0 > 0$ sufficiently small such that $I(u,v) \geqslant r_0$ for $||(u,v)|| = \rho$.

(ii) Let $(u,v) \in X\backslash\{(0,0)\}$, we have

$$
\begin{aligned}
I(tu,tv) &= \frac{at^p}{p}||(u,v)||^p + \frac{bt^{p(k+1)}}{p(k+1)}\Big[(\int_\Omega |\nabla u|^p dx)^{k+1} + (\int_\Omega |\nabla v|^p dx)^{k+1}\Big] \\
&\quad - \frac{t^{\alpha+\beta}}{\alpha+\beta}\int_\Omega |u|^\alpha |v|^\beta dx - \frac{t^q}{q}\int_\Omega (g(x)|u|^q + h(x)|v|^q) dx \\
&\leqslant \frac{at^p}{p}||(u,v)||^p + \frac{bt^{p(k+1)}}{p(k+1)}||(u,v)||^{p(k+1)} - \frac{t^{\alpha+\beta}}{\alpha+\beta}\int_\Omega |u|^\alpha |v|^\beta dx
\end{aligned}
$$

$$-\frac{t^q}{q}\int_\Omega(g(x)|u|^q+h(x)|v|^q)dx$$

Since $p<p(k+1)<q<\alpha+\beta$, we have $I(tu,tv)\to-\infty$ as $t\to+\infty$. Then, for fixed $(u,v)\in X\backslash\{(0,0)\}$, there exists $\bar{t}>0$ such that $||(\bar{t}u,\bar{t}v)||>\rho$ and $I(\bar{t}u,\bar{t}v)<0$. Let $(\bar{u},\bar{v})=(\bar{t}u,\bar{t}v)$, then we finish the proof. □

LEMMA 3. *There exists a $(PS)_\theta$-sequence $\{(u_n,v_n)\}\subset N$ for I.*

Proof. By Lemma 2, I satisfies the conditions of the Mountain Pass Lemma. Thus, by applying Ekeland's variational principle and using the same argument as in Cao and Zhou [8] or Tarantello [5], we can easily find a $(PS)_\theta$-sequence $\{(u_n,v_n)\}\subset N$ for the functional I. □

Next, we will show that I satisfies the $(PS)_\theta$-condition in X.

LEMMA 4. *Let $\{(u_n,v_n)\}\subset X$ be an arbitrary $(PS)_\theta$-sequence for I. That is, $I(u_n,v_n)\to\theta$ and $I'(u_n,v_n)\to 0$ in X^{-1}. Then $\{(u_n,v_n)\}$ has a convergent subsequence.*

Proof. First, we prove that $\{(u_n,v_n)\}$ is bounded in X. In fact, we have

$$\begin{aligned}
\theta+c_n+&\frac{d_n||(u_n,v_n)||}{q}\\
&\geqslant I(u_n,v_n)-\frac{1}{q}\langle I'(u_n,v_n),(u_n,v_n)\rangle\\
&=\frac{a}{p}||(u_n,v_n)||^p+\frac{b}{p(k+1)}\Big[(\int_\Omega|\nabla u_n|^pdx)^{k+1}+(\int_\Omega|\nabla v_n|^pdx)^{k+1}\Big]\\
&\quad-\frac{1}{\alpha+\beta}\int_\Omega|u_n|^\alpha|v_n|^\beta dx-\frac{1}{q}\int_\Omega(g(x)|u_n|^q+h(x)|v_n|^q)dx\\
&\quad-\frac{a}{q}||(u_n,v_n)||^p-\frac{b}{q}\Big[(\int_\Omega|\nabla u_n|^pdx)^{k+1}+(\int_\Omega|\nabla v_n|^pdx)^{k+1}\Big]\\
&\quad+\frac{1}{q}\int_\Omega|u_n|^\alpha|v_n|^\beta dx+\frac{1}{q}\int_\Omega(g(x)|u_n|^q+h(x)|v_n|^q)dx\\
&=(\frac{a}{p}-\frac{a}{q})||(u_n,v_n)||^p+(\frac{b}{p(k+1)}-\frac{b}{q})\Big[(\int_\Omega|\nabla u_n|^pdx)^{k+1}\\
&\quad+(\int_\Omega|\nabla v_n|^pdx)^{k+1}\Big]+(\frac{1}{q}-\frac{1}{\alpha+\beta})\int_\Omega|u_n|^\alpha|v_n|^\beta dx\\
&\geqslant\frac{aq-ap}{pq}||(u_n,v_n)||^p
\end{aligned}$$

where $c_n=o_n(1)$, $d_n=o_n(1)$ as $n\to\infty$. From where we get $\{(u_n,v_n)\}$ is bounded in X. Then, there exist a subsequence (still denoted by $\{(u_n,v_n)\}$) and $(u,v)\in X$ such that

$$u_n\rightharpoonup u, v_n\rightharpoonup v \qquad \text{weakly in } W_0^{1,p}(\Omega);$$

$$u_n \to u, v_n \to v \qquad \text{a.e in } \Omega;$$
$$u_n \to u, v_n \to v \text{ strongly in } L^s(\Omega), \ 1 \leqslant s < p^*$$

and $I'(u,v) = 0$ in X^{-1}.

Next, we prove that

$$\int_\Omega g(x)|u_n|^{q-2}u_n(u_n - u)dx \to 0, \quad n \to \infty \tag{2.1}$$

$$\int_\Omega h(x)|v_n|^{q-2}v_n(v_n - v)dx \to 0, \quad n \to \infty \tag{2.2}$$

and

$$\int_\Omega |u_n|^{\alpha-2}u_n(u_n - u)|v_n|^\beta dx \to 0, \quad n \to \infty \tag{2.3}$$

$$\int_\Omega |v_n|^{\beta-2}v_n(v_n - v)|u_n|^\alpha dx \to 0, \quad n \to \infty \tag{2.4}$$

By Hölder's Inequality, we have

$$\int_\Omega g(x)|u_n|^{q-1}|u_n - u|dx \leqslant C|g|_{\gamma+\delta}|u_n|_{p^*}^{q-1}|u_n - u|_\mu$$

where $\frac{1}{\gamma+\delta} + \frac{q-1}{p^*} + \frac{1}{\mu} = 1$. It is easy to show that $\mu < p^*$ and so $|u_n - u|_\mu \to 0$ as $n \to \infty$.

Since $\{(u_n, v_n)\} \subset X$ is bounded, there exists $M_1 > 0$ such that

$$|u_n|_{p^*}^{q-1} \leqslant S^{-\frac{q-1}{p}}\Big(\int_\Omega |\nabla u_n|^p dx\Big)^{\frac{q-1}{p}} \leqslant S^{-\frac{q-1}{p}}||(u_n, v_n)||^{q-1} \leqslant M_1$$

Then, we can get (2.1). (2.2) can be proved similarly.

By Hölder's Inequality again, we get

$$\int_\Omega |u_n|^{\alpha-1}|u_n - u||v_n|^\beta dx \leqslant |u_n|_{p^*}^{\alpha-1}|v_n|_{p^*}^\beta|u_n - u|_\eta$$

here $\frac{\alpha-1}{p^*} + \frac{\beta}{p^*} + \frac{1}{\eta} = 1$, then $\eta < p^*$ and $|u_n - u|_\eta \to 0$ as $n \to \infty$. Similarly, there exists $M_2, M_3 > 0$ such that $|u_n|_{p^*}^{\alpha-1} \leqslant M_2$, $|v_n|_{p^*}^\beta \leqslant M_3$ Then,

$$\int_\Omega |u_n|^{\alpha-2}u_n(u_n - u)|v_n|^\beta dx \to 0$$

Similarly,

$$\int_\Omega |v_n|^{\beta-2}v_n(v_n - v)|u_n|^\alpha dx \to 0$$

Finally, we prove $||(u_n - u, v_n - v)|| \to 0$ in X. In fact,

$$M\Big(\int_\Omega |\nabla u_n|^p dx\Big)\int_\Omega \langle |\nabla u_n|^{p-2}\nabla u_n - |\nabla u|^{p-2}\nabla u, \nabla u_n - \nabla u\rangle dx$$

$$
\begin{aligned}
&+M\Big(\int_\Omega |\nabla v_n|^p dx\Big)\int_\Omega \langle |\nabla v_n|^{p-2}\nabla v_n - |\nabla v|^{p-2}\nabla v, \nabla v_n - \nabla v\rangle dx \\
&= \langle I'(u_n,v_n),(u_n-u,v_n-v)\rangle - M\Big(\int_\Omega |\nabla u_n|^p dx\Big)\int_\Omega |\nabla u|^{p-2}\langle \nabla u, \nabla u_n - \nabla u\rangle dx \\
&\quad - M\Big(\int_\Omega |\nabla v_n|^p dx\Big)\int_\Omega |\nabla v|^{p-2}\langle \nabla v, \nabla v_n - \nabla v\rangle dx \\
&\quad + \frac{\alpha}{\alpha+\beta}\int_\Omega |u_n|^{\alpha-2}u_n(u_n-u)|v_n|^\beta dx \\
&\quad + \frac{\beta}{\alpha+\beta}\int_\Omega |u_n|^{\alpha}|v_n|^{\beta-2}v_n(v_n-v) dx \\
&\quad + \int_\Omega [g(x)|u_n|^{q-2}u_n(u_n-u) + h(x)|v_n|^{q-2}v_n(v_n-v)]dx.
\end{aligned}
$$

Since $u_n \rightharpoonup u$, $v_n \rightharpoonup v$, we have

$$
\int_\Omega |\nabla u|^{p-2}\langle \nabla u, \nabla u_n - \nabla u\rangle dx \to 0, \quad n\to\infty
$$

$$
\int_\Omega |\nabla v|^{p-2}\langle \nabla v, \nabla v_n - \nabla v\rangle dx \to 0, \quad n\to\infty.
$$

Thus,

$$
\begin{aligned}
&M\Big(\int_\Omega |\nabla u_n|^p dx\Big)\int_\Omega \langle |\nabla u_n|^{p-2}\nabla u_n - |\nabla u|^{p-2}\nabla u, \nabla u_n - \nabla u\rangle dx \\
&\quad + M\Big(\int_\Omega |\nabla v_n|^p dx\Big)\int_\Omega \langle |\nabla v_n|^{p-2}\nabla v_n - |\nabla v|^{p-2}\nabla v, \nabla v_n - \nabla v\rangle dx \to 0.
\end{aligned}
$$

Using the standard inequality

$$
\langle |x|^{p-2}x - |y|^{p-2}y, x-y\rangle \geqslant C_p|x-y|^p, \quad p \geqslant 2
$$

or

$$
\langle |x|^{p-2}x - |y|^{p-2}y, x-y\rangle \geqslant \frac{C_p|x-y|^2}{(|x|+|y|)^{2-p}}, \quad 2>p>1
$$

We obtain

$$
\begin{aligned}
&M\Big(\int_\Omega |\nabla u_n|^p dx\Big)\int_\Omega \langle |\nabla u_n|^{p-2}\nabla u_n - |\nabla u|^{p-2}\nabla u, \nabla u_n - \nabla u\rangle dx \\
&\qquad\qquad \geqslant aC_p\int_\Omega |\nabla u_n - \nabla u|^p dx
\end{aligned}
$$

and

$$
\begin{aligned}
&M\Big(\int_\Omega |\nabla v_n|^p dx\Big)\int_\Omega \langle |\nabla v_n|^{p-2}\nabla v_n - |\nabla v|^{p-2}\nabla v, \nabla v_n - \nabla v\rangle dx \\
&\qquad\qquad \geqslant aC_p\int_\Omega |\nabla v_n - \nabla v|^p dx
\end{aligned}
$$

which implies that

$$\int_{\Omega}(|\nabla u_n-\nabla u|^p+|\nabla v_n-\nabla v|^p)dx\to 0,\quad n\to\infty$$

That is, $||(u_n-u,v_n-v)||\to 0$ in X. Then, the proof is finished. □

3. Proof of Theorem 1

By Lemma 3, there is a minimizing sequence $\{(u_n,v_n)\}\subset N$ for I satisfying $I(u_n,v_n)=\theta+o_n(1)$ and $I'(u_n,v_n)=o_n(1)$ in X^{-1}. By Lemma 4, there exist a subsequence (still denoted by $\{(u_n,v_n)\}$) and $(u_0,v_0)\in X$ such that $(u_n,v_n)\to(u_0,v_0)$ in X. It is easy to show that (u_0,v_0) is a nontrivial solution of (1.1) and $I(u_0,v_0)=\theta$. Using the fact that $I(u_0,v_0)=I(|u_0|,|v_0|)$ and $(|u_0|,|v_0|)\in N$, we may assume that $u_0\geqslant 0$, $v_0\geqslant 0$. By the maximum principle, we can get that $u_0>0$, $v_0>0$ in Ω. Then, (1.1) admits a positive solution.

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