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CORRIGENDUM TO: ANALYSIS AND SOLUTION OF COMPLEX ORDER DIFFERENTIAL EQUATIONS USING SINGULAR KERNEL, PUBLISHED IN DIFFERENTIAL EQUATIONS AND APPLICATIONS, VOL. 16 NO. 2 (2024), 171–182, BY AMAR DEEP

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(*Communicated by M. Fečkan*)

Abstract. Key updates are introduced in this corrigendum: existence findings are presented using upper and lower solutions together with Schauder's fixed point theorem in a new Section 5; a precise solution and verification are added to the original case. Clarifying comments on assumptions in complex-order models have been included, and corrections to the convergence analysis and examples are provided for consistency. The original findings are strengthened and made clearer by these changes.

1. Introduction

In our previous work [4], the theorem numbering contained an inconsistency: Theorem 3 appeared twice; the second occurrence should have been labeled Theorem 5. This has been clarified in the present corrigendum. In addition, a more general existence framework is provided in Section 5 using upper and lower solutions together with Schauder's fixed point theorem. Minor corrections have also been made in the convergence analysis, including the inclusion of the initial term in equation (4.1) and clarification of the recursive inequality in equation (4.4). Due to the addition of Section 5, the original Applications section is renumbered as Section 6. These updates improve the clarity and consistency of the original results. For standard convergence details in Section 4 (see [3]).

5. Existence and uniqueness for the NL-CODEs (3.1)

In this section, existence for problem (3.1) is established using upper and lower solutions and Schauder's fixed point theorem. The uniqueness condition from the original paper [4] is restated for clarity and consistency. Also, the proof follows a different analytical approach from that in [5].

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Let $\mathscr{X} = C(\mathscr{I}, \mathbb{R})$ be a Banach space with norm

$$\|\lambda\| = \|\lambda\|_\infty = \sup_{(\alpha,\beta)\in\mathscr{I}} |\lambda(\alpha,\beta)|.$$

Assume that $\mathscr{H} \in C(\mathscr{I} \times \mathbb{R}^5, \mathbb{R})$.

Define the fixed point operator $\Sigma : \mathscr{X} \to \mathscr{X}$ associated with (3.1) by

$$\Sigma\lambda(\alpha,\beta) = {}_0^{RL}I_\beta^\zeta\big(\mathscr{H}\big(\alpha,\beta,\lambda,\lambda_\beta,\lambda_\alpha,\lambda_{\alpha\alpha},\delta\big)\big), \qquad (\alpha,\beta)\in\mathscr{I}. \tag{5.1}$$

$[W1]$: Let $\lambda^*, \lambda_* \in \mathscr{X}$ such that $\lambda_*(\alpha,\beta) \leqslant \lambda^*(\alpha,\beta)$ for all $(\alpha,\beta)\in\mathscr{I}$ and

$$\begin{aligned} {}_0^{RL}I_\beta^\zeta\big(\mathscr{H}(\alpha,\beta,\lambda^*,\lambda^*_\beta,\lambda^*_\alpha,\lambda^*_{\alpha\alpha},\delta)\big) &\leqslant \lambda^*(\alpha,\beta),\\ {}_0^{RL}I_\beta^\zeta\big(\mathscr{H}(\alpha,\beta,\lambda_*,\lambda_{*\beta},\lambda_{*\alpha},\lambda_{*\alpha\alpha},\delta)\big) &\geqslant \lambda_*(\alpha,\beta). \end{aligned} \tag{5.2}$$

Then λ^* and λ_* are called the upper and lower solutions of (3.1).

THEOREM 1. *Assume that condition* $[W1]$ *is satisfied. Then the NL-CODEs* (3.1) *has at least one solution* $\lambda \in \mathscr{X}$ *satisfying*

$$\lambda_*(\alpha,\beta) \leqslant \lambda(\alpha,\beta) \leqslant \lambda^*(\alpha,\beta), \quad (\alpha,\beta)\in\mathscr{I}.$$

Proof. Let $\mathscr{K} = \{\lambda \in \mathscr{X} : \lambda_*(\alpha,\beta) \leqslant \lambda(\alpha,\beta) \leqslant \lambda^*(\alpha,\beta),\ (\alpha,\beta)\in\mathscr{I}\}$, with norm $\|\lambda\| = \sup_{(\alpha,\beta)\in\mathscr{I}}|\lambda(\alpha,\beta)|$. Then $\mathscr{K}$ is convex, bounded, and closed in the Banach space $\mathscr{X}$. The continuity of $\mathscr{H}$ ensures that the operator Σ (defined in (5.1)) is also continuous on $\mathscr{X}$.

Now, if $\lambda \in \mathscr{X}$, then and there exists a constant $\mathscr{L}' > 0$ such that

$$\big|\mathscr{H}(\alpha,\beta,\lambda,\lambda_\beta,\lambda_\alpha,\lambda_{\alpha\alpha},\delta)\big| \leqslant \mathscr{L}' \text{ for all } (\alpha,\beta)\in\mathscr{I} \text{ and } \quad \|\Sigma\lambda\| \leqslant \frac{\mathscr{L}'}{\Gamma(\zeta_1+1)}.$$

Hence $\Sigma(\mathscr{K})$ is uniformly bounded. Next, we prove the equicontinuity of $\Sigma(\mathscr{K})$. Let $\lambda \in \mathscr{K}$ and $0 \leqslant \beta_1 \leqslant \beta_2 \leqslant 1$ such that $|\beta_1 - \beta_2| \leqslant \delta$.

For further results, let the given hypothesis hold:

$[W2]$: There exist positive constants $L_i > 0$, $i = 1,2,3,4$, such that

$$\big|\mathscr{H}(\alpha,\beta,\lambda_1,\lambda_{1\beta},\lambda_{1\alpha},\lambda_{1\alpha\alpha},\delta) - \mathscr{H}(\alpha,\beta,\lambda_2,\lambda_{2\beta},\lambda_{2\alpha},\lambda_{2\alpha\alpha},\delta)\big| \leqslant \sum_{i=1}^4 L_i|\lambda_1-\lambda_2|.$$

$[W3]$: There exist constants $A_{\mathscr{H}}, B_{\mathscr{H}}, C_{\mathscr{H}} > 0$ such that

$$\begin{aligned} &\big|\mathscr{H}(\alpha,\beta_1,\lambda,\lambda_{\beta_1},\lambda_\alpha,\lambda_{\alpha\alpha},\delta) - \mathscr{H}(\alpha,\beta_2,\lambda,\lambda_{\beta_2},\lambda_\alpha,\lambda_{\alpha\alpha},\delta)\big| \\ &\qquad \leqslant A_{\mathscr{H}} + B_{\mathscr{H}}|\lambda| + C_{\mathscr{H}}|\delta|. \end{aligned} \tag{5.3}$$

$$
\begin{aligned}
&|\Sigma\lambda(\alpha,\beta_1)-\Sigma\lambda(\alpha,\beta_2)| \\
&\quad \leqslant {}_0^{RL}I_\beta^{\zeta_1}\Big(\big|\mathscr{H}(\alpha,\beta_1,\lambda,\lambda_{\beta_1},\lambda_\alpha,\lambda_{\alpha\alpha},\delta)-\mathscr{H}(\alpha,\beta_2,\lambda,\lambda_{\beta_2},\lambda_\alpha,\lambda_{\alpha\alpha},\delta)\big|\Big). \quad (5.4)
\end{aligned}
$$

which implies that

$$
\big\|\Sigma\lambda(\alpha,\beta_1)-\Sigma\lambda(\alpha,\beta_2)\big\| \leqslant \Big(\frac{A_{\mathscr{H}}+B_{\mathscr{H}}\|\lambda\|+C_{\mathscr{H}}\|\delta\|}{\Gamma(\zeta_1+1)}\Big)|\beta_2^{\zeta_1}-\beta_1^{\zeta_1}|. \quad (5.5)
$$

From (5.5), we see that if $\beta_1 \to \beta_2$, then the right-hand side tends to zero. This implies that $|\Sigma\lambda(\alpha,\beta_1)-\Sigma\lambda(\alpha,\beta_2)| \to 0$ as $\beta_1 \to \beta_2$. Therefore, $\Sigma(\mathscr{K})$ is equicontinuous. The Arzelà–Ascoli Theorem implies that $\Sigma : \mathscr{K} \to \mathscr{X}$ is compact. The only thing left to prove is that $\Sigma(\mathscr{K}) \subseteq \mathscr{K}$. Assume that $\mathscr{H}$ is nondecreasing with respect to λ. Let $\lambda \in \mathscr{K}$, then by hypothesis, we have

$$
\begin{aligned}
\Sigma\lambda(\alpha,\beta) &\leqslant {}_0^{RL}I_\beta^{\zeta}\big(\mathscr{H}(\alpha,\beta,\lambda^*(\alpha,\beta),\lambda_\beta^*(\alpha,\beta),\lambda_\alpha^*(\alpha,\beta),\lambda_{\alpha\alpha}^*(\alpha,\beta),\delta(\alpha,\beta))\big) \\
&\leqslant \lambda^*(\alpha,\beta), \quad (5.6)
\end{aligned}
$$

and

$$
\begin{aligned}
\Sigma\lambda(\alpha,\beta) &\geqslant {}_0^{RL}I_\beta^{\zeta}\big(\mathscr{H}(\alpha,\beta,\lambda_*(\alpha,\beta),\lambda_{*\beta}(\alpha,\beta),\lambda_{*\alpha}(\alpha,\beta),\lambda_{*\alpha\alpha}(\alpha,\beta),\delta(\alpha,\beta))\big) \\
&\geqslant \lambda_*(\alpha,\beta). \quad (5.7)
\end{aligned}
$$

Thus,

$$
\lambda_*(\alpha,\beta) \leqslant (\Sigma\lambda)(\alpha,\beta) \leqslant \lambda^*(\alpha,\beta), \quad (\alpha,\beta) \in \mathscr{I},
$$

that is, $\Sigma(\mathscr{K}) \subseteq \mathscr{K}$. According to the Schauder fixed point theorem, the operator Σ has at least one fixed point $\lambda \in \mathscr{K}$. Therefore, the NL-CODEs (3.1) has at least one solution $\lambda \in \mathscr{X}$. The final result concerns the uniqueness of the solution of the NL-CODEs (3.1), which can be established using the Banach contraction principle. □

THEOREM 2. *Assume that condition* $[W1]$ *is satisfied and*

$$
\Big[\sum_{i=1}^{4}\frac{L_i}{\Gamma(\zeta_1+1)}\Big] < 1
$$

holds. Then the proposed problem (3.1) *has a unique solution* $\lambda \in \mathscr{X}$.

REMARK. This result corresponds to Theorem 3 in the original paper [4]. The numbering has been updated due to the addition of new results in Section 5.

Proof. The proof is identical to that of Theorem 3 in [4], and is therefore omitted. □

6. Applications

In [4], TSADM was applied to complex order differential equations, while [5] considered the fractional-order case. In the present corrigendum, Examples 1 and 2 are revised to ensure consistency with the prescribed boundary conditions, and to clarify the applicability of the TSADM approach.

In Example 1, the exact solution is corrected to

$$\lambda(\alpha,\beta) = \beta^{2.7}\alpha^{3.4}(1-\alpha),$$

which satisfies the boundary conditions. Accordingly, the source term $\mathscr{F}(\alpha,\beta)$ is modified so that the corrected exact solution satisfies equation (5.1).

EXAMPLE 2. (TSADM Failure Case)

Consider the NL-CODE [2] given by

$$\begin{aligned} &{}_0^C D_\beta^\zeta \lambda = \cos(\lambda) + 2\sin(\alpha)\lambda_\alpha + \lambda_{\alpha\alpha}\lambda_\beta + \delta + \mathscr{F}(\alpha,\beta), \\ &\lambda(\alpha,0) = 0 = \lambda_\beta(\alpha,0), \\ &\lambda(0,\beta) = 0 = \lambda(1,\beta). \end{aligned} \tag{6.1}$$

To ensure compatibility with the boundary conditions on $[0,1]$, we consider the exact solution $\lambda(\alpha,\beta) = \beta^3\sin(\pi\alpha)$. The source term $\mathscr{F}(\alpha,\beta)$ is chosen so that this function satisfies equation (6.1).

When $\mathscr{F}(\alpha,\beta) = 0$ and the prescribed initial data are trivial, the initial components of the TSADM decomposition vanish, and the recursive scheme produces only the trivial series solution. In such cases, TSADM becomes ineffective. The classical ADM formulation may then be employed by expressing λ and the nonlinear terms as infinite series with the corresponding Adomian polynomials, yielding a nontrivial approximation iteratively. This example highlights that while TSADM performs efficiently under suitable initial data, ADM remains applicable when the decomposition structure of TSADM is not activated, demonstrating the flexibility of decomposition-based methods.

REMARK 1. These examples indicate that the effectiveness of TSADM depends on the availability of suitable initial data that activate the decomposition process. When such conditions are not satisfied, the recursive scheme may yield only the trivial solution. In such cases, alternative formulations such as the classical ADM or MADM can be employed to obtain meaningful approximate solutions. For the previously considered model in [2], once the boundary conditions are correctly imposed, the ADM-based approaches remain effective.

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