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APPLICATIONS OF JS–PREŠIĆ FIXED POINT THEOREM TO A SYSTEM OF NONLINEAR FRACTIONAL INTEGRAL EQUATIONS VIA MEASURE OF NONCOMPACTNESS

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Abstract. This paper presents an extension of the well-known fixed point theorem of Darbo in a Banach space. Using the Prešić type fixed point theorem, which is a generalization of Darbo's fixed point theorem, we investigate the existence of solutions of a system of nonlinear fractional integral equations. Additionally, a suitable example has been given to illustrate the applicability of our findings.

1. Introduction

In several real-world applications across various domains, integral equations are highly beneficial. The measure of noncompactness (MNC) is essential to many fields of research and engineering due to its many uses. An essential component of fixed point (FP) theory is MNC. Many researchers studied the idea of MNC after Kuratowski and Hausdorff's generalization in order to derive important expansions of the theory of *compact operators*. The main area of expertise is applying MNC to ensure that the mappings fulfill the particular inequalities. To help the reader grasp our situations and objective, we provide some context. We go over the basic FP problem in a BS (*Banach space*) $\bar{\mathscr{H}}$ using some assumptions from Schauder [2].

THEOREM 1.1. [10] *In a* BS*, a continuous* operator $\mathtt{T}:\mathtt{S}\to\mathtt{S}$ *admits at least one* FP *for a* convex, nonempty *and* compact *subset* S.

It is the Brouwer FPT (fixed point theorem) generalization.

This paper's format will be as follows: We start by going over some fundamental concepts and terminology associated with FP theory. The FPT is then demonstrated using a newly defined contraction and MNC. Finally, using our results, we examine the existence of solutions of a system of nonlinear FIE.

Using MNC in BS, the authors used Prešić type extension of Darbo FPT to investigate the existence of solution for a system of functional integral equations (FIES) in

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[16]. Additionally, an example is being explored for numerical verification. The authors of [18] examined the solvability of FIES in BS using MNC and Petryshyn's FPT. They also investigated the general class of functional equations, which include several integral equations. The authors of [15] examine the existence of solutions to infinite systems of differential equations (ISDE) of second-order in the space l_p using MNC and a Darbo-type FPT. The authors of [17] work on existence results for an ISDE of order n in the spaces c_0 and l_1 with boundary conditions using a method related to MNC. In [13], the authors showed that there is a solution for an infinite system of nonlinear integral equations in the BS l_p, $p > 1$ using a technique associated with MNC and the generalized Meir-Keeler FPT. In [12], the authors investigated whether nonlinear integral equations have any solutions. They also offered an iterative method to solve the nonlinear integral equations with high accuracy. Finally, they gave an upper bound of error and established the convergence condition.

Inspired by these research works, we are going to generalize a Prešić type FPT from Darbo FPT, and we look into the existence of solutions of a system of nonlinear FIE.

2. Preliminaries

We collect some basic symbols, definitions that are required for the paper as follows. Suppose $\overline{conv}(I)$ and $\bar{I}$ (for all nonempty set I) denotes the smallest convex, closed set containing I and the closure of I respectively.

Moreover $\check{\mathrm{V}}_{\bar{\mathscr{H}}}$, $\check{\mathrm{U}}_{\bar{\mathscr{H}}}$ represents the set of bounded, nonempty subsets of $\bar{\mathscr{H}}$ and the set of $\bar{\mathscr{H}}$ including all relatively *compact* and nonempty sets respectively. $\tilde{\mathscr{R}}_+ = [0,\infty)$; $\tilde{\mathscr{R}} = (-\infty,\infty)$, and N denotes the natural numbers set.

DEFINITION 2.1. [2] A map $\tilde{\mathtt{G}} : \check{\mathrm{V}}_{\bar{\mathscr{H}}} \to \tilde{\mathscr{R}}_+$ is a MNC in $\bar{\mathscr{H}}$, if

1. $ker\{\tilde{\mathtt{G}}\} = \{\check{\mathrm{E}} \in \check{\mathrm{V}}_{\bar{\mathscr{H}}} : \tilde{\mathtt{G}}(\check{\mathrm{E}}) = 0\} \neq \phi$,
2. $\check{\mathrm{E}}_1 \subseteq \check{\mathrm{E}}_2 \Rightarrow \tilde{\mathtt{G}}(\check{\mathrm{E}}_1) \leqslant \tilde{\mathtt{G}}(\check{\mathrm{E}}_2)$,
3. $\tilde{\mathtt{G}}\left(\overline{\check{\mathrm{E}}}\right) = \tilde{\mathtt{G}}(\check{\mathrm{E}})$,
4. $\tilde{\mathtt{G}}(\overline{conv}(\check{\mathrm{E}})) = \tilde{\mathtt{G}}(\check{\mathrm{E}})$,
5. $\tilde{\mathtt{G}}\left(\breve{\beta}\check{\mathrm{E}}_1 + \left(1-\breve{\beta}\right)\check{\mathrm{E}}_2\right) \leqslant \breve{\beta}\tilde{\mathtt{G}}(\check{\mathrm{E}}_1) + \left(1-\breve{\beta}\right)\tilde{\mathtt{G}}(\check{\mathrm{E}}_2)$, for $\breve{\beta} \in [0,1]$,
6. The set $\check{\mathrm{E}}_\infty = \cap_{n=1}^{\infty}\check{\mathrm{E}}_n$ is non empty, if $\{\check{\mathrm{E}}_n\}$ is a sequence of closed sets with $\check{\mathrm{E}}_{n+1} \subset \check{\mathrm{E}}_n$ in $\check{\mathrm{V}}_{C^{n,\acute{\delta}}}$ and $\lim_{n\to\infty} \tilde{\mathtt{G}}(\check{\mathrm{E}}_n) = 0$, $n \in N$.

THEOREM 2.2. [16] *Assume $\bar{\mathscr{H}}$ be a* BS, *and a* NBCCS *(nonempty, bounded, closed, convex subset) $\tilde{\mathfrak{D}}$ of $\bar{\mathscr{H}}$. A map $\hat{\mathtt{T}} : \tilde{\mathfrak{D}} \to \tilde{\mathfrak{D}}$, which is continuous and compact admits atleast a* FP.

THEOREM 2.3. [16] *For a* BS $\bar{\mathscr{H}}$, *consider a* NBCCS $\bar{\mathbb{S}}$. *Assume a map* $\hat{\mathtt{T}}:\bar{\mathbb{S}}\to\bar{\mathbb{S}}$, *which is continuous and* $\exists$ $\check{b}\in[0,1)$, *with*

$$\tilde{\mathtt{G}}(\hat{\mathtt{T}}(\mathscr{Q}))\leqslant\check{b}\tilde{\mathtt{G}}(\mathscr{Q}),$$

for $\mathscr{Q}\subseteq\bar{\mathbb{S}}$, *and* $\tilde{\mathtt{G}}$ *is the* MNC *in the space* $\bar{\mathscr{H}}$. *Atleast a* FP *is then admitted by* $\hat{\mathtt{T}}$.

According to the well-established BCMP (*Banach contraction mapping principle*) [5], if $\tilde{\Theta}:\hat{\mathtt{J}}\to\hat{\mathtt{J}}$ is a map to itself, for a CMS(complete metric space) $(\hat{\mathtt{J}},\tilde{\Theta})$ in such a way that

$$d(\tilde{\Theta}\acute{\upsilon},\tilde{\Theta}\acute{\mathtt{e}})\leqslant\bar{\mathtt{w}}d(\acute{\upsilon},\acute{\mathtt{e}}),$$

for all $\acute{\upsilon},\acute{\mathtt{e}}\in\hat{\mathtt{J}}$, then $\exists$ a unique $\check{\beta}$ with $\check{\beta}=\tilde{\Theta}(\check{\beta})$, where $1>\bar{\mathtt{w}}\geqslant0$. Numerous generalizations of this idea have surfaced in recent years. Prešić developed the following result.

THEOREM 2.4. [16] *For a* CMS $(\hat{\mathtt{J}},\tilde{\Theta})$, *consider a map* $\tilde{\Theta}:\hat{\mathtt{J}}^n\to\hat{\mathtt{J}}$. *Assume that*

$$d(\tilde{\Theta}(\acute{\upsilon}_1,\acute{\upsilon}_2,\ldots,\acute{\upsilon}_n),\tilde{\Theta}(\acute{\upsilon}_2,\acute{\upsilon}_3,\ldots,\acute{\upsilon}_{n+1}))\leqslant\sum_{\mathtt{k}=1}^n\bar{\mathtt{y}}_{\mathtt{k}}d(\acute{\upsilon}_{\mathtt{k}},\acute{\upsilon}_{\mathtt{k}+1}),$$

for all $\acute{\upsilon}_1,\acute{\upsilon}_2,\ldots,\acute{\upsilon}_{n+1}\in\hat{\mathtt{J}}$ *where* $0\leqslant\sum_{\mathtt{k}=1}^n\bar{\mathtt{y}}_{\mathtt{k}}<1$, n *is a positive integer, and* $\bar{\mathtt{y}}_{\mathtt{k}}\geqslant0$. *Then* $\tilde{\Theta}$ *admits a unique* FP $\tilde{\check{\beta}}$.

Moreover, the sequence $\acute{\upsilon}_n$ defined as $\acute{\upsilon}_{n+\mathtt{k}}=\tilde{\Theta}(\acute{\upsilon}_n,\acute{\upsilon}_{n+1},\ldots,\acute{\upsilon}_{n+\mathtt{k}-1})$, converges to $\tilde{\check{\beta}}$, for all arbitrary points $\acute{\upsilon}_1,\acute{\upsilon}_2,\ldots,\acute{\upsilon}_{n+1}\in\hat{\mathtt{J}}$.

Theorem 2.4 coincides with the Banach contraction principle, if we consider $n=1$.

The aforementioned theorem was generalized as follows by Prešić and Ćirić:

THEOREM 2.5. [16] *For a* CMS $(\hat{\mathtt{J}},\tilde{\Theta})$, *consider a map* $\tilde{\Theta}:\hat{\mathtt{J}}^n\to\hat{\mathtt{J}}$. *Assume that*

$$d(\tilde{\Theta}(\acute{\upsilon}_1,\acute{\upsilon}_2,\ldots,\acute{\upsilon}_n),\tilde{\Theta}(\acute{\upsilon}_2,\acute{\upsilon}_3,\ldots,\acute{\upsilon}_{n+1}))\leqslant\bar{\mathtt{w}}\max\{d(\acute{\upsilon}_{\mathtt{k}},\acute{\upsilon}_{\mathtt{k}+1}):\mathtt{k}\in[1,n]\},$$

for all $\acute{\upsilon}_1,\acute{\upsilon}_2,\ldots,\acute{\upsilon}_{n+1}\in\hat{\mathtt{J}}$ *where* $1>\bar{\mathtt{w}}\geqslant0$. *Then* $\tilde{\Theta}$ *admits a fixed point* $\tilde{\check{\beta}}\in\hat{\mathtt{J}}$.

Moreover, the sequence $\acute{\upsilon}_n$ defined as $\acute{\upsilon}_{n+\mathtt{k}}=\tilde{\Theta}(\acute{\upsilon}_n,\acute{\upsilon}_{n+1},\ldots,\acute{\upsilon}_{n+\mathtt{k}-1})$, converges to $\tilde{\check{\beta}}$, for all arbitrary points $\acute{\upsilon}_1,\acute{\upsilon}_2,\ldots,\acute{\upsilon}_{n+1}\in\hat{\mathtt{J}}$.

If for all $\bar{\mathtt{p}},\check{\mathtt{I}}\in\hat{\mathtt{J}}$, $\bar{\mathtt{p}}\neq\check{\mathtt{I}}$,

$$d(\tilde{\Theta}(\bar{\mathtt{p}},\bar{\mathtt{p}},\ldots,\bar{\mathtt{p}}),\tilde{\Theta}(\check{\mathtt{I}},\check{\mathtt{I}},\ldots,\check{\mathtt{I}}))<d(\bar{\mathtt{p}},\check{\mathtt{I}}).$$

Then $\tilde{\Theta}$ admits a unique FP $\tilde{\check{\beta}}\in\hat{\mathtt{J}}$.

3. Main result

DEFINITION 3.1. [8] Consider that $\bar{\mathsf{B}}$ be a *family* of continuous map $\bar{\Lambda}:\mathscr{R}_+\to\mathscr{R}_+$ such that $\bar{\Lambda}(\tilde{p})>\tilde{p},\ \bar{\Lambda}(0)=0$, for $\tilde{p}\in\mathscr{R}_+\setminus\{0\}$.

As an example, we can consider $\bar{\Lambda}(\tilde{p})=\bar{\lambda}\tilde{p}$, for all $\breve{p}\in\mathscr{R}_+,\ \bar{\lambda}>1$.

DEFINITION 3.2. [8] Consider $\bar{\mathsf{S}}$ be a *family* of non-decreasing, continuous maps $\bar{\mathscr{T}}:\mathscr{R}_+\to\mathscr{R}_+$ with $\bar{\mathscr{T}}(\tilde{p})>\tilde{p},\ \tilde{p}\in\mathscr{R}_+$.

As an example, we can consider $\bar{\mathscr{T}}(\tilde{p})=\bar{\mu}\tilde{p}$, for all $\breve{p}\in\mathscr{R}_+,\ \bar{\mu}>1$.

DEFINITION 3.3. [8] Consider $\bar{\mathsf{K}}$ be a *family* of monotonic increasing maps $\tilde{\varpi}:\mathscr{R}_+\to\mathscr{R}_+$ with

$$\lim_{k\to\infty}\tilde{\varpi}^k(\tilde{p})=0.$$

for all $\tilde{p}>0$.

As an example, we can consider $\tilde{\varpi}(\tilde{p})=\bar{\zeta}\tilde{p}$, for all $\breve{p}\in\mathscr{R}_+,\ 0<\bar{\zeta}<1$.

THEOREM 3.4. *Consider $\bar{\mathscr{D}}$ be a* NBCCS *of a* BS $\bar{\mathscr{H}}$ *and a continuous operator $\tilde{\Theta}:\bar{\mathscr{D}}\to\bar{\mathscr{D}}$ with*

$$\bar{\Lambda}\Big(\bar{\mathscr{T}}\Big(\tilde{\mathtt{G}}\Big(\tilde{\Theta}(\hat{\mathtt{J}})\Big)\Big)\Big)\leqslant\tilde{\varpi}\Big(\tilde{\mathtt{G}}(\hat{\mathtt{J}})\Big), \tag{3.1}$$

where $\hat{\mathtt{J}}\subseteq\bar{\mathscr{D}},\ \bar{\Lambda}\in\bar{\mathsf{B}},\ \bar{\mathscr{T}}\in\bar{\mathsf{S}},\ \tilde{\varpi}\in\bar{\mathsf{K}}$ and $\tilde{\mathtt{G}}$ is a MNC *in $\bar{\mathscr{H}}$. Then there exists atleast a fixed point admitted by $\tilde{\Theta}$ on $\bar{\mathscr{D}}$.*

Proof. Consider $\{\mathscr{V}_k\}$ with $\mathscr{V}_k=conv\left(\tilde{\Theta}(\mathscr{V}_{k-1})\right)$ and $\mathscr{V}_0=\bar{\mathscr{D}},\ k\geqslant 1$.

If for some $k\in N,\ \tilde{\mathtt{G}}(\mathscr{V}_k)=0$, then $\mathscr{V}_k$ is relatively compact. By Theorem 2.2, we get that $\tilde{\Theta}$ admits a FP. Hence for all $k\geqslant 0$, assume $\tilde{\mathtt{G}}(\mathscr{V}_k)>0$. Clearly $\{\mathscr{V}_k\}$ is a sequence of NBCCS with

$$\mathscr{V}_0\supseteq\mathscr{V}_1\supseteq\mathscr{V}_2\ldots\supseteq\mathscr{V}_k\supseteq\mathscr{V}_{k+1}.$$

Now we have

$$\begin{aligned}\bar{\Lambda}\Big(\bar{\mathscr{T}}\Big(\tilde{\mathtt{G}}(\mathscr{V}_{k+1})\Big)\Big)&=\bar{\Lambda}\Big(\bar{\mathscr{T}}\Big(\tilde{\mathtt{G}}(conv(\mathfrak{D}(\mathscr{V}_k)))\Big)\Big)\\&=\bar{\Lambda}\Big(\bar{\mathscr{T}}\Big(\tilde{\mathtt{G}}(\mathfrak{D}(\mathscr{V}_k))\Big)\Big)\\&\leqslant\tilde{\varpi}\Big(\tilde{\mathtt{G}}(\mathscr{V}_k)\Big)\\&\leqslant\tilde{\varpi}^2\Big(\tilde{\mathtt{G}}(\mathscr{V}_{k-1})\Big)\\&\ \ \vdots\\&\leqslant\tilde{\varpi}^{k+1}\Big(\tilde{\mathtt{G}}(\mathscr{V}_0)\Big)\end{aligned} \tag{3.2}$$

Now from the definition (3.1) of $\bar{\Lambda}$, we have,

$$\bar{\mathscr{T}}\left(\tilde{\mathtt{G}}\left(\mathscr{V}_{k+1}\right)\right) < \bar{\Lambda}\left(\bar{\mathscr{T}}\left(\tilde{\mathtt{G}}\left(\mathscr{V}_{k+1}\right)\right)\right) \tag{3.3}$$

Using equation (3.2) and (3.3), we get

$$\bar{\mathscr{T}}\left(\tilde{\mathtt{G}}\left(\mathscr{V}_{k+1}\right)\right) < \tilde{\varpi}^{k+1}\left(\tilde{\mathtt{G}}\left(\mathscr{V}_{0}\right)\right) \tag{3.4}$$

Now by definition (3.3) we get

$$\bar{\mathscr{T}}\left(\tilde{\mathtt{G}}\left(\mathscr{V}_{k+1}\right)\right) = 0, \text{ as } \quad k \to \infty.$$

Thus, we have

$$\lim_{k\to\infty} \tilde{\mathtt{G}}(\mathscr{V}_{k+1}) = \lim_{k\to\infty} \tilde{\mathtt{G}}(\mathscr{V}_{k}) = 0.$$

Let $\mathscr{V}_\infty = \cap_{k=0}^{\infty} \mathscr{V}_k$, so we get an element $\mathscr{V}_\infty$ of $ker\tilde{\mathtt{G}}\left(\mathscr{V}_k\right)$, which is NBCCS and invariant under $\tilde{\Theta}$. Hence from Theorem 2.2, $\tilde{\Theta}$ has a FP. □

COROLLARY 3.5. *Consider $\bar{\mathscr{D}}$ be a* NBCCS *of a* BS $\bar{\mathscr{H}}$ *and a continuous operator $\tilde{\Theta} : \bar{\mathscr{D}} \to \bar{\mathscr{D}}$ so that*

$$\tilde{\mathtt{G}}\left(\tilde{\Theta}\left(\hat{\mathtt{J}}\right)\right) \leqslant \bar{\lambda}\tilde{\mathtt{G}}\left(\hat{\mathtt{J}}\right),$$

for $\hat{\mathtt{J}} \subseteq \bar{\mathscr{D}}$, $\bar{\lambda} \in (0,1)$, and $\tilde{\mathtt{G}}$ is a MNC *in $\bar{\mathscr{H}}$. Then there exists atleast a* FP *admitted by $\tilde{\Theta}$ on $\bar{\mathscr{D}}$.*

Proof. Putting $\bar{\Lambda}(\breve{p}) = \bar{w}\breve{p}$, $\bar{\mathscr{T}}(\breve{p}) = \acute{\mathtt{h}}\breve{p}$, and $\tilde{\varpi}(\breve{p}) = \acute{r}\breve{p}$, $\forall\ \breve{p} > 0$, $\acute{r} < 1$, and $\acute{\mathtt{h}}, \bar{w} > 1$ in equation (3.1) of Theorem 3.4, we get the result shown above. □

THEOREM 3.6. *Consider $\bar{\mathscr{D}}$ be a* NBCCS *of a* BS $\bar{\mathscr{H}}$ *and a continuous operator $\tilde{\Theta} : \bar{\mathscr{D}}^n \to \bar{\mathscr{D}}$ so that*

$$\bar{\Lambda}\left(\bar{\mathscr{T}}\left(\tilde{\mathtt{G}}\left(\tilde{\Theta}\left(\hat{\mathtt{J}}_1 \times \hat{\mathtt{J}}_2 \times \ldots \times \hat{\mathtt{J}}_n\right)\right)\right)\right) \leqslant \tilde{\varpi}\left(\max\left\{\tilde{\mathtt{G}}\left(\hat{\mathtt{J}}_1\right), \ldots, \tilde{\mathtt{G}}\left(\hat{\mathtt{J}}_n\right)\right\}\right), \tag{3.5}$$

$\hat{\mathtt{J}}_1, \hat{\mathtt{J}}_2, \ldots, \hat{\mathtt{J}}_n \subseteq \bar{\mathscr{D}}$, $\bar{\Lambda} \in \bar{\mathsf{B}}$, $\bar{\mathscr{T}} \in \bar{\mathsf{S}}$, $\tilde{\varpi} \in \bar{\mathsf{K}}$ *and $\tilde{\mathtt{G}}$ is a* MNC *in $\bar{\mathscr{H}}$. Then there exists atleast a Prešić type* FP *admitted by $\tilde{\Theta}$ on $\bar{\mathscr{D}}$.*

Proof. For $\acute{\zeta}_1, \acute{\zeta}_2, \ldots, \acute{\zeta}_n \in \bar{\mathscr{D}}$, define a map $\Theta : \bar{\mathscr{D}}^n \to \bar{\mathscr{D}}^n$ as

$$\Theta\left(\acute{\zeta}_1, \acute{\zeta}_2, \ldots, \acute{\zeta}_n\right) = \left(\tilde{\Theta}\left(\acute{\zeta}_1, \acute{\zeta}_2, \ldots, \acute{\zeta}_n\right), \ldots, \tilde{\Theta}\left(\acute{\zeta}_1, \acute{\zeta}_2, \ldots, \acute{\zeta}_n\right)\right).$$

Clearly Θ is continuous and all conditions of theorem satisfies by Θ.

Moreover $\mathtt{G}(\hat{\daleth}) = \max\left\{\tilde{\mathtt{G}}(\hat{\daleth}_1), \ldots, \tilde{\mathtt{G}}(\hat{\daleth}_n)\right\}$ is a MNC [16], $\hat{\daleth} \subset \bar{\mathscr{D}}^n$, $\hat{\daleth}_1, \hat{\daleth}_2, \ldots, \hat{\daleth}_n \subset \bar{\mathscr{D}}$.

For a nonempty subset $\hat{\daleth} \subset \bar{\mathscr{D}}^n$, we have

$$
\begin{aligned}
&\bar{\Lambda}\left(\bar{\mathscr{T}}\left(\mathtt{G}\left(\Theta(\hat{\daleth})\right)\right)\right)\\
&= \bar{\Lambda}\left(\bar{\mathscr{T}}\left(\mathtt{G}\left(\tilde{\Theta}(\hat{\daleth}_1 \times \hat{\daleth}_2 \times \ldots \times \hat{\daleth}_n), \ldots, \tilde{\Theta}(\hat{\daleth}_1 \times \hat{\daleth}_2 \times \ldots \times \hat{\daleth}_n)\right)\right)\right)\\
&= \bar{\Lambda}\left(\bar{\mathscr{T}}\left(\max\left\{\tilde{\mathtt{G}}\left(\tilde{\Theta}(\hat{\daleth}_1 \times \hat{\daleth}_2 \times \ldots \times \hat{\daleth}_n)\right), \ldots, \tilde{\mathtt{G}}\left(\tilde{\Theta}(\hat{\daleth}_1 \times \hat{\daleth}_2 \times \ldots \times \hat{\daleth}_n)\right)\right\}\right)\right)\\
&= \bar{\Lambda}\left(\bar{\mathscr{T}}\left(\tilde{\mathtt{G}}\left(\tilde{\Theta}(\hat{\daleth}_1 \times \hat{\daleth}_2 \times \ldots \times \hat{\daleth}_n)\right)\right)\right)\\
&\leqslant \tilde{\varpi}\left(\max\left\{\tilde{\mathtt{G}}(\hat{\daleth}_1), \ldots, \tilde{\mathtt{G}}(\hat{\daleth}_n)\right\}\right)\\
&= \tilde{\varpi}\left(\mathtt{G}(\hat{\daleth})\right).
\end{aligned}
$$

Hence from Theorem 3.4, Θ has atleast a FP, which implies that atleast a Prešić type FP admitted by $\tilde{\Theta}$ on $\bar{\mathscr{D}}$. □

COROLLARY 3.7. *Consider* $\bar{\mathscr{D}}$ *be a* NBCCS *of a* BS $\bar{\mathscr{H}}$ *and a continuous operator* $\tilde{\Theta} : \bar{\mathscr{D}}^n \to \bar{\mathscr{D}}$ *so that*

$$
\tilde{\mathtt{G}}\left(\tilde{\Theta}(\hat{\daleth}_1 \times \hat{\daleth}_2 \times \ldots \times \hat{\daleth}_n)\right) \leqslant \bar{\lambda}\left(\max\left\{\tilde{\mathtt{G}}(\hat{\daleth}_1), \ldots, \tilde{\mathtt{G}}(\hat{\daleth}_n)\right\}\right), \tag{3.6}
$$

$\hat{\daleth}_1, \hat{\daleth}_2, \ldots, \hat{\daleth}_n \subset \bar{\mathscr{D}}$, $\bar{\lambda} \in (0,1)$, *and* $\tilde{\mathtt{G}}$ *is a* MNC *in* $\bar{\mathscr{H}}$. *Then there exists atleast a Prešić type* FP *admitted by* $\tilde{\Theta}$ *on* $\bar{\mathscr{D}}$.

Proof. Putting $\bar{\Lambda}(\breve{p}) = \bar{w}\breve{p}$, $\bar{\mathscr{T}}(\breve{p}) = \acute{\mathtt{h}}\breve{p}$, and $\tilde{\varpi}(\breve{p}) = \acute{r}\breve{p}$, for all $\acute{\mathtt{h}}, \breve{p} > 0$, $\acute{r} < 1$, and $\bar{w} > 1$ in equation (3.5) of Theorem 3.6, we obtain the result shown above. □

4. Applications

We apply our findings to investigate the existence of solution the following nonlinear FIE in the space $\mathtt{BC}(\mathscr{R}_+)$:

$$
\begin{cases}
\bar{\beth}_1(\acute{\omega}) = \Xi(\acute{\omega}) + \frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}} \Delta\left(\acute{\omega}, \acute{\upsilon}, \bar{\beth}_1(\hat{\theta}(\acute{\omega})), \bar{\beth}_2(\hat{\theta}(\acute{\omega})), \ldots, \bar{\beth}_n(\hat{\theta}(\acute{\omega}))\right) d\acute{\upsilon}\\
\bar{\beth}_2(\acute{\omega}) = \Xi(\acute{\omega}) + \frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}} \Delta\left(\acute{\omega}, \acute{\upsilon}, \bar{\beth}_1(\hat{\theta}(\acute{\omega})), \bar{\beth}_2(\hat{\theta}(\acute{\omega})), \ldots, \bar{\beth}_n(\hat{\theta}(\acute{\omega}))\right) d\acute{\upsilon}\\
\vdots\\
\bar{\beth}_n(\acute{\omega}) = \Xi(\acute{\omega}) + \frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}} \Delta\left(\acute{\omega}, \acute{\upsilon}, \bar{\beth}_1(\hat{\theta}(\acute{\omega})), \bar{\beth}_2(\hat{\theta}(\acute{\omega})), \ldots, \bar{\beth}_n(\hat{\theta}(\acute{\omega}))\right) d\acute{\upsilon},
\end{cases} \tag{4.1}
$$

where $\bar{\beth}_1, \bar{\beth}_2, \ldots, \bar{\beth}_n \in \mathtt{BC}(\mathscr{R}_+)$; $\acute{\alpha} \in \mathscr{R}_+$; $\acute{\upsilon}, \acute{\omega} \in [0, \bar{\mathscr{B}}]$, $\bar{\mathscr{B}} > 0$.

Also $\Xi : \mathscr{R}_+ \to \mathscr{R}$, $\Delta : \mathscr{R}_+ \times \mathscr{R}_+ \times \mathscr{R}^n \to \mathscr{R}$ and $\hat{\theta} : \mathscr{R}_+ \to \mathscr{R}_+$ are continuous functions, and $\Gamma(,)$ denotes Euler's gamma function.

Consider

$$\bar{H}_{\mathfrak{M}_0} = \{\tilde{u} \in \mathtt{BC}(\mathscr{R}_+) : \|\tilde{u}\| \leqslant \mathfrak{M}_0\},$$

for a constant $\mathfrak{M}_0 > 0$.

Here $\|\tilde{u}\| = \sup\{|\tilde{u}(\acute{\omega})| : \acute{\omega} \geqslant 0\}$ denotes the norm in the space $\mathtt{BC}(\mathscr{R}_+)$.

Also define $\tilde{\mathscr{M}}^{\tilde{\mathscr{B}}}(\Upsilon, \bar{\kappa}) = \sup\{|\Upsilon(\tilde{u}_1) - \Upsilon(\tilde{u}_2)| : \tilde{u}_1, \tilde{u}_2 \in [0, \bar{\mathscr{B}}], |\tilde{u}_1 - \tilde{u}_2| \leqslant \bar{\kappa}\}$ as the modulus of continuity of a function $\Upsilon \in \mathtt{BC}(\mathscr{R}_+)$.

Consider $\tilde{\mathscr{M}}^{\tilde{\mathscr{B}}}(\hat{\beth}, \bar{\kappa}) = \sup\{\tilde{\mathscr{M}}^{\tilde{\mathscr{B}}}(\Upsilon, \bar{\kappa}) : \Upsilon \in \hat{\beth}\}$,

$$\tilde{\mathscr{M}}_0^{\tilde{\mathscr{B}}}(\hat{\beth}) = \lim_{\bar{\kappa} \to 0} \tilde{\mathscr{M}}^{\tilde{\mathscr{B}}}(\hat{\beth}, \bar{\kappa}),$$

and

$$\tilde{\mathscr{M}}_0(\hat{\beth}) = \lim_{\tilde{\mathscr{B}} \to \infty} \tilde{\mathscr{M}}_0^{\tilde{\mathscr{B}}}(\hat{\beth}).$$

Assume

$$\hat{\beth}(\acute{\omega}) = \{\Upsilon(\acute{\omega}) : \Upsilon \in \hat{\beth}\},$$

and for fixed $\acute{\omega} \in \mathscr{R}_+$,

$$diam\hat{\beth}(\acute{\omega}) = \sup\{|\bar{\beth}_1(\acute{\omega}) - \bar{\beth}_2(\acute{\omega})| : \bar{\beth}_1, \bar{\beth}_2 \in \hat{\beth}\},$$

The $\mathtt{MNC}$ in the space $\mathtt{BC}(\mathscr{R}_+)$ defined as [3]

$$\tilde{\mathscr{G}}_0(\hat{\beth}) = \tilde{\mathscr{M}}_0(\hat{\beth}) + \lim_{\acute{\omega} \to \infty} \sup diam\hat{\beth}(\acute{\omega}).$$

In order to examine the solutions of the system of equation (4.1), we now make the following assumptions.

(i) $\Xi : \mathscr{R}_+ \to \mathscr{R}$ is a continuous map satisfying,

$$|\Xi(\acute{\omega}_1) - \Xi(\acute{\omega}_2)| \leqslant |\acute{\omega}_1 - \acute{\omega}_2|,$$

for $\acute{\omega}_1, \acute{\omega}_2 \in \mathscr{R}_+$.

(ii) $\Delta : \mathscr{R}_+ \times \mathscr{R}_+ \times \mathscr{R}^n \to \mathscr{R}$ is a continuous map satisfying,

$$\left|\Delta\Big(\acute{\omega}_1, \acute{\upsilon}_1, \bar{u}_1, \bar{u}_2, \ldots, \bar{u}_n\Big) - \Delta\Big(\acute{\omega}_2, \acute{\upsilon}_2, \hat{\varsigma}_1, \hat{\varsigma}_2, \ldots, \hat{\varsigma}_n\Big)\right|$$
$$\leqslant |\acute{\omega}_1 - \acute{\omega}_2| + |\acute{\upsilon}_1 - \acute{\upsilon}_2| + \max\{|\bar{u}_1 - \hat{\varsigma}_1|, |\bar{u}_2 - \hat{\varsigma}_2|, \ldots, |\bar{u}_n - \hat{\varsigma}_n|\}$$

for $\acute{\omega}_1, \acute{\omega}_2, \acute{\upsilon}_1, \acute{\upsilon}_2 \in \mathscr{R}_+$, and $\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_n, \hat{\varsigma}_1, \hat{\varsigma}_2, \ldots, \hat{\varsigma}_n \in \mathtt{BC}(\mathscr{R}_+)$.

(iii)

$$\tilde{\mathscr{N}} = \sup\{|\Xi(\acute{\omega})| : \acute{\omega} \in \mathscr{R}_+\},$$
$$\tilde{\mathscr{Q}} = \sup\left\{\left|\Delta\Big(\acute{\omega}, \acute{\upsilon}, 0, 0, \ldots, 0\Big)\right| : \acute{\omega}, \acute{\upsilon} \in \mathscr{R}_+\right\}$$
$$\bar{\mathbb{K}} = \sup\left\{\left|\Delta\Big(\acute{\omega}_1, \acute{\upsilon}, \bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}_1\right)\right), \bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}_1\right)\right), \ldots, \bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)\Big)\right| : \acute{\omega}_1, \acute{\omega}_2, \acute{\upsilon} \in \mathscr{R}_+\right\}$$

for each $\bar{\beth}_1, \bar{\beth}_2, \ldots, \bar{\beth}_n \in \mathtt{BC}(\mathscr{R}_+)$; and $\hat{\theta} : \mathscr{R}_+ \to \mathscr{R}_+$ is a continuous function.

Also

$$\left|\Delta\Big(\acute{\omega}_1, \acute{\upsilon}, \bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}_1\right)\right), \bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}_1\right)\right), \ldots, \bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)\Big)\right| \leqslant \bar{\mathsf{A}}(\acute{\omega}_1)\mathsf{H}(\acute{\upsilon}),$$

with $\lim\limits_{\acute{\omega}_1 \to \infty} \int_0^{\acute{\omega}_1} \bar{\mathsf{A}}(\acute{\omega}_1)\mathsf{H}(\acute{\upsilon})\, d\acute{\upsilon} = 0$, for two continuous function $\bar{\mathsf{A}}, \mathsf{H}$ and $\acute{\omega}_1, \acute{\upsilon} \in \mathscr{R}_+$, $\bar{\beth}_1, \bar{\beth}_2, \ldots, \bar{\beth}_n \in \mathtt{BC}(\mathscr{R}_+)$.

(iv) $\exists$ a $\mathfrak{M}_0 > 0$, for each $\bar{\beth}_1, \bar{\beth}_2, \ldots, \bar{\beth}_n \in \mathtt{BC}(\mathscr{R}_+)$; $\acute{\alpha} \in \mathscr{R}_+$; $\acute{\upsilon}, \acute{\omega} \in [0, \bar{\mathscr{B}}]$, $\bar{\mathscr{B}} > 0$, and $\hat{\theta}, \tilde{\mathscr{N}}, \tilde{\mathscr{Q}}$ are as defined above satisfying

$$\tilde{\mathscr{N}} + \frac{1}{\Gamma(\acute{\alpha})} \int_0^{\acute{\omega}} \max\left\{|\bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}\right)\right)|, |\bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}\right)\right)|, \ldots, |\bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}\right)\right)|\right\} d\acute{\upsilon} + \frac{\acute{\omega}}{\Gamma(\acute{\alpha})} \tilde{\mathscr{Q}}$$
$$\leqslant \mathfrak{M}_0.$$

THEOREM 4.1. *If conditions (i)–(iv) holds and $\frac{\bar{\mathscr{B}}}{\Gamma(\acute{\alpha})} < \bar{\zeta}$, $\bar{\zeta} \in (0,1)$. Then the system of equations* (4.1) *has atleast a solution in* $\mathtt{BC}(\mathscr{R}_+)$.

Proof. First we define the function $\tilde{\Theta} : \mathtt{BC}(\mathscr{R}_+) \times \ldots \times \mathtt{BC}(\mathscr{R}_+) \longrightarrow \mathtt{BC}(\mathscr{R}_+)$, for an arbitrary fixed $\acute{\omega} > 0$ as

$$\tilde{\Theta}\left(\bar{\beth}_1, \bar{\beth}_2, \ldots, \bar{\beth}_n\right)(\acute{\omega})$$
$$= \Xi(\acute{\omega}) + \frac{1}{\Gamma(\acute{\alpha})} \int_0^{\acute{\omega}} \Delta\Big(\acute{\omega}, \acute{\upsilon}, \bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}\right)\right), \bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}\right)\right), \ldots, \bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}\right)\right)\Big)\, d\acute{\upsilon}$$

Now, we prove that the operator $\tilde{\Theta}$ maps from $\left(\bar{H}_{\mathfrak{M}_0}\right)^n$ into $\bar{H}_{\mathfrak{M}_0}$.

For $\acute{\omega} > 0$, we have

$$\begin{aligned}
&|\tilde{\Theta}\left(\bar{\beth}_1,\bar{\beth}_2,\ldots,\bar{\beth}_n\right)(\acute{\omega})| \\
&= \left|\Xi(\acute{\omega}) + \frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}} \Delta\Big(\acute{\omega},\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega})\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega})\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega})\right)\Big)\,d\acute{\upsilon}\right| \\
&\leqslant |\Xi(\acute{\omega})| + \left|\frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}} \Big\{\Delta\Big(\acute{\omega},\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega})\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega})\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega})\right)\Big)\right. \\
&\quad \left. - \Delta\Big(\acute{\omega},\acute{\upsilon},0,0,\ldots,0\Big) + \Delta\Big(\acute{\omega},\acute{\upsilon},0,0,\ldots,0\Big)\Big\}\,d\acute{\upsilon}\right| \\
&\leqslant \tilde{\mathscr{N}} + \frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}} \max\Big\{|\bar{\beth}_1\left(\hat{\theta}(\acute{\omega})\right)|,|\bar{\beth}_2\left(\hat{\theta}(\acute{\omega})\right)|,\ldots,|\bar{\beth}_n\left(\hat{\theta}(\acute{\omega})\right)|\Big\}\,d\acute{\upsilon} + \frac{\acute{\omega}}{\Gamma(\acute{\alpha})}\mathscr{Q} \\
&\leqslant \mathfrak{M}_0.
\end{aligned}$$

Thus, $\tilde{\Theta}$ maps from $(\bar{H}_{\mathfrak{M}_0})^n$ into $\bar{H}_{\mathfrak{M}_0}$.

Now, we prove that $\tilde{\Theta}$ is a continuous map. Assume $(\bar{u}_1,\bar{u}_2,\ldots,\bar{u}_n),(\hat{\varsigma}_1,\hat{\varsigma}_2,\ldots,\hat{\varsigma}_n) \in (\bar{H}_{\mathfrak{M}_0})^n$ with $\|\bar{u}_1-\hat{\varsigma}_1\|+\|\bar{u}_2-\hat{\varsigma}_2\|+\ldots+\|\bar{u}_n-\hat{\varsigma}_n\| < \bar{\kappa}$, for arbitrary fix $\bar{\kappa} > 0$. Then we have for all $\acute{\omega} \in \mathscr{R}_+$,

$$\begin{aligned}
&|\tilde{\Theta}(\bar{u}_1,\bar{u}_2,\ldots,\bar{u}_n)(\acute{\omega}) - \tilde{\Theta}(\hat{\varsigma}_1,\hat{\varsigma}_2,\ldots,\hat{\varsigma}_n)(\acute{\omega})| \\
&\leqslant \left|\Xi(\acute{\omega}) + \frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}} \Delta\Big(\acute{\omega},\acute{\upsilon},\bar{u}_1\left(\hat{\theta}(\acute{\omega})\right),\bar{u}_2\left(\hat{\theta}(\acute{\omega})\right),\ldots,\bar{u}_n\left(\hat{\theta}(\acute{\omega})\right)\Big)\,d\acute{\upsilon} - \Xi(\acute{\omega})\right. \\
&\quad \left. - \frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}} \Delta\Big(\acute{\omega},\acute{\upsilon},\hat{\varsigma}_1\left(\hat{\theta}(\acute{\omega})\right),\hat{\varsigma}_2\left(\hat{\theta}(\acute{\omega})\right),\ldots,\hat{\varsigma}_n\left(\hat{\theta}(\acute{\omega})\right)\Big)\,d\acute{\upsilon}\right| \\
&\leqslant \left|\frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}} \Big\{\Delta\Big(\acute{\omega},\acute{\upsilon},\bar{u}_1\left(\hat{\theta}(\acute{\omega})\right),\bar{u}_2\left(\hat{\theta}(\acute{\omega})\right),\ldots,\bar{u}_n\left(\hat{\theta}(\acute{\omega})\right)\Big)\right. \\
&\quad \left. - \Delta\Big(\acute{\omega},\acute{\upsilon},\hat{\varsigma}_1\left(\hat{\theta}(\acute{\omega})\right),\hat{\varsigma}_2\left(\hat{\theta}(\acute{\omega})\right),\ldots,\hat{\varsigma}_n\left(\hat{\theta}(\acute{\omega})\right)\Big)\Big\}\,d\acute{\upsilon}\right| \\
&\leqslant \frac{\bar{\mathscr{B}}}{\Gamma(\acute{\alpha})}\ \max\Big\{|\bar{u}_1\left(\hat{\theta}(\acute{\omega})\right)-\hat{\varsigma}_1\left(\hat{\theta}(\acute{\omega})\right)|,|\bar{u}_2\left(\hat{\theta}(\acute{\omega})\right)-\hat{\varsigma}_2\left(\hat{\theta}(\acute{\omega})\right)|, \\
&\quad \ldots,|\bar{u}_n\left(\hat{\theta}(\acute{\omega})\right)-\hat{\varsigma}_n\left(\hat{\theta}(\acute{\omega})\right)|\Big\} \\
&\leqslant \bar{\zeta}\ \max\Big\{|\bar{u}_1\left(\hat{\theta}(\acute{\omega})\right)-\hat{\varsigma}_1\left(\hat{\theta}(\acute{\omega})\right)|,|\bar{u}_2\left(\hat{\theta}(\acute{\omega})\right)-\hat{\varsigma}_2\left(\hat{\theta}(\acute{\omega})\right)|, \\
&\quad \ldots,|\bar{u}_n\left(\hat{\theta}(\acute{\omega})\right)-\hat{\varsigma}_n\left(\hat{\theta}(\acute{\omega})\right)|\Big\}
\end{aligned}$$

Here $\|\bar{u}_1-\hat{\varsigma}_1\|+\|\bar{u}_2-\hat{\varsigma}_2\|+\ldots+\|\bar{u}_n-\hat{\varsigma}_n\| < \bar{\kappa}$.

So, as $\bar{\kappa} \to 0$, $|\tilde{\Theta}(\bar{u}_1,\bar{u}_2,\ldots,\bar{u}_n)(\acute{\omega}) - \tilde{\Theta}(\hat{\varsigma}_1,\hat{\varsigma}_2,\ldots,\hat{\varsigma}_n)(\acute{\omega})| \to 0$.

Thus, $\tilde{\Theta}$ is continuous.

Next, for fixed $\acute{\omega} > 0$, and a sequence $\{\acute{\omega}_n\}$ such that $\acute{\omega}_n \to \acute{\omega}$ as $n \to \infty$. We can select $\acute{\omega}_n \geqslant \acute{\omega}$ without losing generality. Then

$$
\begin{aligned}
&|\tilde{\Theta}\left(\bar{\beth}_1,\bar{\beth}_2,\ldots,\bar{\beth}_n\right)(\acute{\omega}_n)-\tilde{\Theta}\left(\bar{\beth}_1,\bar{\beth}_2,\ldots,\bar{\beth}_n\right)(\acute{\omega})|\\
&=\Bigg|\Xi(\acute{\omega}_n)+\frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}_n}\Delta\Big(\acute{\omega}_n,\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega}_n)\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega}_n)\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega}_n)\right)\Big)\,d\acute{\upsilon}\\
&\quad-\Xi(\acute{\omega})-\frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}}\Delta\Big(\acute{\omega},\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega})\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega})\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega})\right)\Big)\,d\acute{\upsilon}\Bigg|\\
&\leqslant\Big|\Xi(\acute{\omega}_n)-\Xi(\acute{\omega})\Big|\\
&\quad+\Bigg|\frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}_n}\Big\{\Delta\Big(\acute{\omega}_n,\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega}_n)\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega}_n)\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega}_n)\right)\Big)\\
&\quad-\Delta\Big(\acute{\omega},\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega})\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega})\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega})\right)\Big)\Big\}\,d\acute{\upsilon}\Bigg|\\
&\quad+\Bigg|\frac{1}{\Gamma(\acute{\alpha})}\int_{\acute{\omega}}^{\acute{\omega}_n}\Delta\Big(\acute{\omega},\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega})\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega})\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega})\right)\Big)\,d\acute{\upsilon}\Bigg|\\
&\leqslant|\acute{\omega}_n-\acute{\omega}|+\frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}_n}\Big\{|\acute{\omega}_n-\acute{\omega}|+\max\Big\{|\bar{\beth}_1\left(\hat{\theta}(\acute{\omega}_n)\right)-\bar{\beth}_1\left(\hat{\theta}(\acute{\omega})\right)|,\\
&\quad|\bar{\beth}_2\left(\hat{\theta}(\acute{\omega}_n)\right)-\bar{\beth}_2\left(\hat{\theta}(\acute{\omega})\right)|,\ldots,|\bar{\beth}_n\left(\hat{\theta}(\acute{\omega}_n)\right)-\bar{\beth}_n\left(\hat{\theta}(\acute{\omega})\right)|\Big\}\Big\}\,d\acute{\upsilon}+\frac{\bar{\mathbb{K}}}{\Gamma(\acute{\alpha})}|\acute{\omega}_n-\acute{\omega}|.
\end{aligned}
$$

Thus, $|\tilde{\Theta}\left(\bar{\beth}_1,\bar{\beth}_2,\ldots,\bar{\beth}_n\right)(\acute{\omega}_n)-\tilde{\Theta}\left(\bar{\beth}_1,\bar{\beth}_2,\ldots,\bar{\beth}_n\right)(\acute{\omega})|\to 0$, as $n\to\infty$.

Next, assume $\acute{\omega}_1,\acute{\omega}_2\in[0,\bar{\mathscr{B}}]$ with $|\acute{\omega}_2-\acute{\omega}_1|\leqslant\bar{\kappa},\ 0<\bar{\mathscr{B}}$, and $(\bar{\beth}_1,\bar{\beth}_2,\ldots,\bar{\beth}_n)\in\hat{\beth}_1\times\ldots\times\hat{\beth}_n$, Then

$$
\begin{aligned}
&|\tilde{\Theta}\left(\bar{\beth}_1,\bar{\beth}_2,\ldots,\bar{\beth}_n\right)(\acute{\omega}_2)-\tilde{\Theta}\left(\bar{\beth}_1,\bar{\beth}_2,\ldots,\bar{\beth}_n\right)(\acute{\omega}_1)|\\
&=\Bigg|\Xi(\acute{\omega}_2)+\frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}_2}\Delta\Big(\acute{\omega}_2,\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega}_2)\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega}_2)\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega}_2)\right)\Big)\,d\acute{\upsilon}\\
&\quad-\Xi(\acute{\omega}_1)-\frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}_1}\Delta\Big(\acute{\omega}_1,\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega}_1)\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega}_1)\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega}_1)\right)\Big)\,d\acute{\upsilon}\Bigg|\\
&\leqslant\Big|\Xi(\acute{\omega}_2)-\Xi(\acute{\omega}_1)\Big|\\
&\quad+\Bigg|\frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}_2}\Big\{\Delta\Big(\acute{\omega}_2,\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega}_2)\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega}_2)\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega}_2)\right)\Big)\\
&\quad-\Delta\Big(\acute{\omega}_1,\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega}_1)\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega}_1)\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega}_1)\right)\Big)\Big\}\,d\acute{\upsilon}\Bigg|\\
&\quad+\Bigg|\frac{1}{\Gamma(\acute{\alpha})}\int_{\acute{\omega}_1}^{\acute{\omega}_2}\Delta\Big(\acute{\omega}_1,\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}(\acute{\omega}_1)\right),\bar{\beth}_2\left(\hat{\theta}(\acute{\omega}_1)\right),\ldots,\bar{\beth}_n\left(\hat{\theta}(\acute{\omega}_1)\right)\Big)\,d\acute{\upsilon}\Bigg|
\end{aligned}
$$

$$
\begin{aligned}
&\leqslant |\acute{\omega}_2-\acute{\omega}_1| + \frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}_2}\Big\{|\acute{\omega}_2-\acute{\omega}_1| + \max\Big\{|\bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}_2\right)\right) - \bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)|, \\
&\quad |\bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}_2\right)\right) - \bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)|, \ldots, |\bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}_2\right)\right) - \bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)|\Big\}\Big\}\, d\acute{\upsilon} \\
&\quad + \frac{\bar{\mathbb{K}}}{\Gamma(\acute{\alpha})}|\acute{\omega}_2-\acute{\omega}_1| \\
&\leqslant |\acute{\omega}_2-\acute{\omega}_1| + \frac{\bar{\mathscr{B}}}{\Gamma(\acute{\alpha})}|\acute{\omega}_2-\acute{\omega}_1| \\
&\quad + \frac{\bar{\mathscr{B}}}{\Gamma(\acute{\alpha})}\ \max\Big\{|\bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}_2\right)\right) - \bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)|, |\bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}_2\right)\right) - \bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)|, \\
&\quad \ldots, |\bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}_2\right)\right) - \bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)|\Big\} + \frac{\bar{\mathbb{K}}}{\Gamma(\acute{\alpha})}|\acute{\omega}_2-\acute{\omega}_1|.
\end{aligned}
$$

Now,

$$
\begin{aligned}
&\tilde{\mathscr{M}}^{\bar{\mathscr{B}}}(\tilde{\Theta}(\hat{\gimel}_1 \times \ldots \times \hat{\gimel}_n), \bar{\kappa}) \\
&\leqslant |\acute{\omega}_2-\acute{\omega}_1| + \frac{\bar{\mathscr{B}}}{\Gamma(\acute{\alpha})}|\acute{\omega}_2-\acute{\omega}_1| \\
&\quad + \frac{\bar{\mathscr{B}}}{\Gamma(\acute{\alpha})}\ \max\Big\{|\bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}_2\right)\right) - \bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)|, |\bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}_2\right)\right) - \bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)|, \\
&\quad \ldots, |\bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}_2\right)\right) - \bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)|\Big\} + \frac{\bar{\mathbb{K}}}{\Gamma(\acute{\alpha})}|\acute{\omega}_2-\acute{\omega}_1|
\end{aligned}
$$

Also,

$$
\begin{aligned}
\tilde{\mathscr{M}}_0^{\bar{\mathscr{B}}}\Big(\tilde{\Theta}\left(\hat{\gimel}_1, \hat{\gimel}_2, \ldots, \hat{\gimel}_n\right)\Big) &\leqslant \frac{\bar{\mathscr{B}}}{\Gamma(\acute{\alpha})}\ \max\Big\{\tilde{\mathscr{M}}_0^{\bar{\mathscr{B}}}\left(\hat{\gimel}_1\right), \tilde{\mathscr{M}}_0^{\bar{\mathscr{B}}}\left(\hat{\gimel}_2\right), \ldots, \tilde{\mathscr{M}}_0^{\bar{\mathscr{B}}}\left(\hat{\gimel}_n\right)\Big\} \\
&\leqslant \bar{\zeta}\ \max\Big\{\tilde{\mathscr{M}}_0^{\bar{\mathscr{B}}}\left(\hat{\gimel}_1\right), \tilde{\mathscr{M}}_0^{\bar{\mathscr{B}}}\left(\hat{\gimel}_2\right), \ldots, \tilde{\mathscr{M}}_0^{\bar{\mathscr{B}}}\left(\hat{\gimel}_n\right)\Big\}.
\end{aligned}
$$

Thus

$$
\tilde{\mathscr{M}}_0\Big(\tilde{\Theta}\left(\hat{\gimel}_1, \hat{\gimel}_2, \ldots, \hat{\gimel}_n\right)\Big) \leqslant \bar{\zeta}\ \max\Big\{\tilde{\mathscr{M}}_0\left(\hat{\gimel}_1\right), \tilde{\mathscr{M}}_0\left(\hat{\gimel}_2\right), \ldots, \tilde{\mathscr{M}}_0\left(\hat{\gimel}_n\right)\Big\}.
$$

And

$$
\begin{aligned}
&diam\Big\{\tilde{\Theta}\left(\hat{\gimel}_1, \hat{\gimel}_2, \ldots, \hat{\gimel}_n\right)(\acute{\omega})\Big\} \\
&\leqslant \bar{\zeta}\ \max\Big\{diam\,\hat{\gimel}_1\left(\hat{\theta}\left(\acute{\omega}\right)\right), diam\,\hat{\gimel}_2\left(\hat{\theta}\left(\acute{\omega}\right)\right), \ldots, diam\,\hat{\gimel}_n\left(\hat{\theta}\left(\acute{\omega}\right)\right)\Big\}.
\end{aligned}
$$

Thus

$$\begin{aligned}
&\tilde{\mathscr{G}_0}\Big(\tilde{\Theta}\left(\hat{\beth}_1,\hat{\beth}_2,\ldots,\hat{\beth}_n\right)\Big)\\
&= \tilde{\mathscr{M}_0}\Big(\tilde{\Theta}\left(\hat{\beth}_1,\hat{\beth}_2,\ldots,\hat{\beth}_n\right)\Big) + \limsup_{\acute{\omega}\to\infty} diam\Big\{\tilde{\Theta}\left(\hat{\beth}_1,\hat{\beth}_2,\ldots,\hat{\beth}_n\right)(\acute{\omega})\Big\}\\
&\leqslant \bar{\zeta}\ \max\Big\{\tilde{\mathscr{M}_0}\left(\hat{\beth}_1\right),\tilde{\mathscr{M}_0}\left(\hat{\beth}_2\right),\ldots,\tilde{\mathscr{M}_0}\left(\hat{\beth}_n\right)\Big\}\\
&\quad + \bar{\zeta}\ \max\Big\{diam\,\hat{\beth}_1\left(\hat{\theta}\left(\acute{\omega}\right)\right), diam\,\hat{\beth}_2\left(\hat{\theta}\left(\acute{\omega}\right)\right),\ldots, diam\,\hat{\beth}_n\left(\hat{\theta}\left(\acute{\omega}\right)\right)\Big\}\\
&\leqslant \bar{\zeta}\ \max\Big\{\tilde{\mathscr{G}_0}\left(\hat{\beth}_1\right),\tilde{\mathscr{G}_0}\left(\hat{\beth}_2\right),\ldots,\tilde{\mathscr{G}_0}\left(\hat{\beth}_n\right)\Big\}.
\end{aligned}$$

Therefore, by Corollary 3.7, $\tilde{\Theta}$ has a Prešić type fixed point. Thus in $\mathtt{BC}(\mathscr{R}_+)$, the system of equations (4.1) admits atleast one solution and we are finished. □

EXAMPLE 4.2. Assume the following FIE with $\mathfrak{M}_0 = 6$, $\bar{\mathbb{K}} = 5$,

$$\begin{cases}
\bar{\beth}_1(\acute{\omega}) = \acute{\omega} + \frac{1}{\Gamma\left(\frac{5}{2}\right)}\int_0^{\acute{\omega}}\left(\acute{\omega}+\acute{\upsilon}+\frac{\bar{\beth}_1\left(\hat{\theta}(\acute{\omega})\right)+\bar{\beth}_2\left(\hat{\theta}(\acute{\omega})\right)+\ldots+\bar{\beth}_n\left(\hat{\theta}(\acute{\omega})\right)}{n}\right)d\acute{\upsilon}\\
\bar{\beth}_2(\acute{\omega}) = \acute{\omega} + \frac{1}{\Gamma\left(\frac{5}{2}\right)}\int_0^{\acute{\omega}}\left(\acute{\omega}+\acute{\upsilon}+\frac{\bar{\beth}_1\left(\hat{\theta}(\acute{\omega})\right)+\bar{\beth}_2\left(\hat{\theta}(\acute{\omega})\right)+\ldots+\bar{\beth}_n\left(\hat{\theta}(\acute{\omega})\right)}{n}\right)d\acute{\upsilon}\\
\vdots\\
\bar{\beth}_n(\acute{\omega}) = \acute{\omega} + \frac{1}{\Gamma\left(\frac{5}{2}\right)}\int_0^{\acute{\omega}}\left(\acute{\omega}+\acute{\upsilon}+\frac{\bar{\beth}_1\left(\hat{\theta}(\acute{\omega})\right)+\bar{\beth}_2\left(\hat{\theta}(\acute{\omega})\right)+\ldots+\bar{\beth}_n\left(\hat{\theta}(\acute{\omega})\right)}{n}\right)d\acute{\upsilon},
\end{cases} \tag{4.2}$$

for $\acute{\omega},\acute{\upsilon}\in[0,1]$.

Consider $\max\Big\{|\bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}\right)\right)|,|\bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}\right)\right)|,\ldots,|\bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}\right)\right)|\Big\} \leqslant e^{-\acute{\omega}}$.

Now for $\acute{\omega}_1,\acute{\omega}_2\in[0,1]$, Ξ satisfies,

$$|\Xi(\acute{\omega}_1)-\Xi(\acute{\omega}_2)| = |\acute{\omega}_1-\acute{\omega}_2|.$$

Next, for $\acute{\omega}_1,\acute{\omega}_2,\acute{\upsilon}_1,\acute{\upsilon}_2\in[0,1]$, and $\bar{u}_1,\bar{u}_2,\ldots,\bar{u}_n,\hat{\varsigma}_1,\hat{\varsigma}_2,\ldots,\hat{\varsigma}_n\in\mathtt{BC}(\mathscr{R}_+)$,

$$\begin{aligned}
&\Big|\Delta\Big(\acute{\omega}_1,\acute{\upsilon}_1,\bar{u}_1,\bar{u}_2,\ldots,\bar{u}_n\Big)-\Delta\Big(\acute{\omega}_2,\acute{\upsilon}_2,\hat{\varsigma}_1,\hat{\varsigma}_2,\ldots,\hat{\varsigma}_n\Big)\Big|\\
&= |\acute{\omega}_1+\acute{\upsilon}_1+\frac{\bar{u}_1+\bar{u}_2+\ldots+\bar{u}_n}{n}-\acute{\omega}_2-\acute{\upsilon}_2-\frac{\hat{\varsigma}_1+\hat{\varsigma}_2+\ldots+\hat{\varsigma}_n}{n}|\\
&\leqslant |\acute{\omega}_1-\acute{\omega}_2|+|\acute{\upsilon}_1-\acute{\upsilon}_2|+\frac{1}{n}\{|\bar{u}_1-\hat{\varsigma}_1|+|\bar{u}_2-\hat{\varsigma}_2|+\ldots+|\bar{u}_n-\hat{\varsigma}_n|\}\\
&\leqslant |\acute{\omega}_1-\acute{\omega}_2|+|\acute{\upsilon}_1-\acute{\upsilon}_2|+\max\{|\bar{u}_1-\hat{\varsigma}_1|,|\bar{u}_2-\hat{\varsigma}_2|,\ldots,|\bar{u}_n-\hat{\varsigma}_n|\}
\end{aligned}$$

Also,

$$\begin{aligned}
\tilde{\mathscr{N}} &= \sup\{|\Xi(\acute{\omega})| : \acute{\omega}\in[0,1]\}\\
&= \sup\{|\acute{\omega}| : \acute{\omega}\in[0,1]\}\\
&\leqslant 1.
\end{aligned}$$

$$\begin{aligned}\tilde{\mathscr{Q}} &= \sup\left\{\left|\Delta\Big(\acute{\omega},\acute{\upsilon},0,0,\ldots,0\Big)\right| : \acute{\omega},\acute{\upsilon}\in[0,1]\right\}\\ &= \sup\left\{\left|\acute{\omega}+\acute{\upsilon}\right| : \acute{\omega},\acute{\upsilon}\in[0,1]\right\}\\ &\leqslant 2.\end{aligned}$$

Next

$$\left|\Delta\Big(\acute{\omega}_1,\acute{\upsilon},\bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}_1\right)\right),\bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}_1\right)\right),\ldots,\bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}_1\right)\right)\Big)\right| \leqslant e^{-\acute{\omega}_1}e^{-\acute{\upsilon}},$$

with $\lim_{\acute{\omega}_1\to\infty}\int_0^{\acute{\omega}_1} e^{-\acute{\omega}_1}e^{-\acute{\upsilon}}\,d\acute{\upsilon}=0.$

Also,

$$\begin{aligned}&\tilde{\mathscr{N}}+\frac{1}{\Gamma(\acute{\alpha})}\int_0^{\acute{\omega}}\max\left\{|\bar{\beth}_1\left(\hat{\theta}\left(\acute{\omega}\right)\right)|,|\bar{\beth}_2\left(\hat{\theta}\left(\acute{\omega}\right)\right)|,\ldots,|\bar{\beth}_n\left(\hat{\theta}\left(\acute{\omega}\right)\right)|\right\}d\acute{\upsilon}+\frac{\acute{\omega}}{\Gamma(\acute{\alpha})}\tilde{\mathscr{Q}}\\ &\leqslant 1+\frac{1}{\Gamma\left(\frac{5}{2}\right)}\acute{\omega}e^{-\acute{\omega}}+\frac{\acute{\omega}}{\Gamma\left(\frac{5}{2}\right)}2.\\ &\leqslant 1+\frac{1}{3}\acute{\omega}e^{-\acute{\omega}}+\frac{2\acute{\omega}}{3}.\\ &\leqslant 6.\end{aligned}$$

Since all the assumptions of Theorem 4.1 are satisfied. Hence, we get that the system of equations (4.2) has atleast one solution in the space $\mathtt{BC}(\mathscr{R}_+)$.

Declarations

Data availability. Not applicable.

Conflicts of interests. There are no conflicts of interest, according to the authors.

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REFERENCES

[1] R. R. AKHMEROV, M. I. KAMENSKII, A. S. POTAPOV, A. E. RODKINA, B. N. SADOVSKII, *Measures of Noncompactness and condensing Operators*, vol. 55, Birkhäuser, Basel, 1992.

[2] J. BANAŚ AND K. GOEBEL, *Measure of Noncompactness in Banach Spaces*, Lecture Notes in Pure and Applied Mathematics, vol. 60, Marcel Dekker, New York, 1980.

[3] J. BANAŚ, *Measures of noncompactness in the space of continuous tempered functions*, Demonstratio Math. **14**, 127–133, (1981).

[4] G. S. CHEN, *Mean value theorems for Local fractional integrals on fractal space*, Adv. Mech. Eng. Appl. **1**, 5–8 (2012).

[5] L. B. ĆIRIĆ, S. B. PREŠIĆ, *On Prešić type generalization of the Banach contraction mapping principle*, Acta. Math. Univ. Com. LXXVI, (2) (2007), 143–147.

[6] A. DAS, B. HAZARIKA, P. KUMAM, *Some New Generalization of Darbo's Fixed Point Theorem and Its Application on Integral Equations*, Mathematics 2019, **7** (3), 214, https://doi.org/10.3390/math7030214.

[7] A. DAS, B. HAZARIKA, R. ARAB, M. MURSALEEN, *Applications of a fixed point theorem to the existence of solutions to the nonlinear functional integral equations in two variables*, Rendiconti del circolo Matematico di Palermo, Rend. Circ. Mat. Palermo, II. Ser, https://doi.org/10.1007/s12215-018-0347-9.

[8] S. DEB, A. DAS, *Modified version of fixed point theorems and their applications on a fractional hybrid differential equation in the space of continuous tempered functions*, J. Pseudo-Differ. Oper. Appl. (2023) **14**: 75, https://doi.org/10.1007/s11868-023-00570-2.

[9] K. DIETHELM, *The mean value theorem and a Nagumo-type uniqueness theorem for Caputo's fractional calculus*, Fractional Calculus and Applied Analysis, vol. 15, pages 304–313, (2012), doi:10.2478/s13540-012-0022-3.

[10] M. GABELEH, J. MAKIN, *Global Optimal Solutions of a Differenial Equations via Measure of Noncompactness*, Filomat **35**: 15 (2021), 5059–5071, https:/doi.og/10.2298/FIL2115059G.

[11] M. A. E. HERZALLAH, D. BALEANU, *On Fractional Order Hybrid Differential Equations*, Hindawi Publishing Corporation, Abstract and Applied Analysis, vol. 2014, Article ID 389386, 7 pages, http://dx.doi.org/10.1155/2014/389386.

[12] B. HAZARIKA, H. M. SRIVASTAVA, R. ARAB, M. RABBANI, *Application of simulation function and measure of noncompactness for solvability of nonlinear functional integral equations and introduction to an iteration algorithm to find solution*, Applied Mathematics and Computation, vol. 360, 1 November 2019, pages 131–146, https://doi.org/10.1016/j.amc.2019.04.058.

[13] B. HAZARIKA, H. M. SRIVASTAVA, R. ARAB, M. RABBANI, *Existence of solution for an infinite system of nonlinear integral equations via measure of noncompactness and homotopy perturbation method to solve it*, Journal of Computational and Applied Mathematics, vol. 343, 1 December 2018, pages 341–352, https://doi.org/10.1016/j.cam.2018.05.011.

[14] H. A. KAYVANLOO, M. KHANEHGIR, R. ALLAHYARI, *A family of measure of Noncompactness in the Hölder space $C^{n,\acute{\delta}}\left(\tilde{\mathscr{R}}_+\right)$ and its application to some fractional differential equations and numerical methods*, Journal of computational and applied mathematics, https:/doi.org/10.1016/j.cam.2019.06.012.

[15] S. A. MOHIUDDINE, H. M. SRIVASTAVA AND A. ALOTAIBI, *Application of measures of noncompactness to the infinite system of second-order differential equations in l_p space*, Mohiuddine et al., Advances in Difference Equations (2016) **2016**: 317, doi:10.1186/s13662-016-1016-y.

[16] M. PARVANEHA, A. FARAJZADEHA, *Measure of noncompactness and JS-Prešić fixed point theorems and its applications to a system of integral equations*, Filomat **35**: 9 (2021), 3091–3104, https://doi.org/10.2298/FIL2109091P.

[17] H. M. SRIVASTAVA, A. DAS, B. HAZARIKA, S. A. MOHIUDDINE, *Existence of solutions of infinite systems of differential equations of general order with boundary conditions in the spaces c_0 and l_1 via the measure of noncompactness*, Mathematical Methods in the Applied Sciences **41** (10), 3558–3569, https://doi.org/10.1002/mma.4845.

[18] H. M. SRIVASTAVA, A. DEEP, S. ABBAS, AND B. HAZARIKA, *Solvability for a class of generalized functional-integral equations by means of Petryshyn's fixed point theorem*, Journal of Nonlinear and Convex Analysis, vol. 22, no. 12, 2021, 2715–2737.

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