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SOME PROPERTIES AND INEQUALITIES FOR h–GEOMETRICALLY CONVEX FUNCTIONS

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Abstract. In the paper, the definition of h-geometrically convex functions is introduced, some properties of h-geometrically convex functions are studied, and several integral inequality for the newly defined functions are established.

1. Introduction

Throughout this paper, we use the following notations:

$$\mathbb{R} = (-\infty, \infty), \quad \mathbb{R}_0 = [0, \infty), \quad \text{and} \quad \mathbb{R}_+ = (0, \infty).$$

We first recite some definitions of various convex functions.

DEFINITION 1.1. A function $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is said to be convex if

$$f(tx + (1-t)y) \leqslant tf(x) + (1-t)f(y) \tag{1.1}$$

holds for all $x, y \in I$ and $t \in [0,1]$.

DEFINITION 1.2. A function $f : I \subseteq \mathbb{R}_+ \to \mathbb{R}_+$ is said to be geometrically convex if

$$f\left(x^t y^{1-t}\right) \leqslant [f(x)]^t [f(y)]^{1-t} \tag{1.2}$$

holds for all $x, y \in I$ and $t \in [0,1]$.

DEFINITION 1.3. ([14, Definition 1.9]) A function $f : I \subseteq \mathbb{R}_+ \to \mathbb{R}_+$ is said to be s-geometrically convex if

$$f\left(x^t y^{1-t}\right) \leqslant [f(x)]^{t^s} [f(y)]^{(1-t)^s} \tag{1.3}$$

for some $s \in (0,1]$, where $x, y \in I$ and $t \in [0,1]$.

DEFINITION 1.4. ([6, Definition 4]) Let $I, J \subseteq \mathbb{R}$ be intervals, $(0,1) \subseteq J$, $h : J \to \mathbb{R}_0$ be a non-negative function, and $h \not\equiv 0$. A function $f : I \to \mathbb{R}_0$ is called h-convex, or say, f belongs to the class $\mathrm{SX}(h, I)$, if f is non-negative and

$$f(tx + (1-t)y) \leqslant h(t)f(x) + h(1-t)f(y) \tag{1.4}$$

for all $x, y \in I$ and $t \in [0,1]$.

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DEFINITION 1.5. ([6, Section 3]) Let h be a function defined on an interval $J \subseteq \mathbb{R}$. It is said to be super-multiplicative on J if

$$h(xy) \geqslant h(x)h(y) \tag{1.5}$$

is valid for all $x, y \in J$. If the inequality (1.5) reverses, then f is said to be a sub-multiplicative function on J.

For more information on notions of various convex functions and their Hermite-Hadamard type inequalities, please refer to recently published articles [1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 15] and closely related references therein.

In this paper, we will introduce a new notion "h-geometrically convex function", study some basic properties of this kind of functions, and establish several inequalities of the newly introduced functions.

2. A new notion

We now introduce a new notion "h-geometrically convex function" as follows.

DEFINITION 2.1. Let $h : [0,1] \to \mathbb{R}_0$ and $h \not\equiv 0$. A function $f : I \subseteq \mathbb{R}_+ \to \mathbb{R}_+$ is called h-geometrically convex, or say, f belongs to the class $\mathrm{HGX}(h,I)$, if

$$f\left(x^t y^{1-t}\right) \leqslant [f(x)]^{h(t)}[f(y)]^{h(1-t)} \tag{2.1}$$

for all $x, y \in I$ and $t \in [0,1]$. If the inequality (2.1) is reversed, then f is called h-geometrically concave and denoted by $f \in \mathrm{HGV}(h,I)$.

REMARK 2.1. It is clear that

1. when $h(t) = t^s$ on $[0,1]$, the h-geometrically convex functions become the s-geometrically convex functions;
2. when $h(t) = t$ on $[0,1]$, the h-geometrically convex functions become the geometrically convex functions.

REMARK 2.2. It is easy to show that the inequality (2.1) is equivalent to

$$\ln f\left(e^{t\ln x+(1-t)\ln y}\right) \leqslant h(t)\ln f\left(e^{\ln x}\right) + h(1-t)\ln f\left(e^{\ln y}\right) \tag{2.2}$$

for all $x, y \in I$ and $t \in [0,1]$.

REMARK 2.3. The above Definition 2.1 was also independently introduced in the preprint [4, Definition 9].

3. Properties

We now discuss some properties of the h-geometrically convex functions.

THEOREM 3.1. *Let* $h:[0,1]\to\mathbb{R}_0$ *and* $f:I\subseteq\mathbb{R}_+\to[1,\infty)$. *Then* $f\in\mathrm{HGX}(h,I)$ *if and only if* $\ln f(e^u)\in\mathrm{SX}(h,\ln I)$, *where* $\ln I\triangleq\{\ln x : x\in I\}$.

Proof. This follows from the inequality (2.2). □

THEOREM 3.2. *Let* $h:[0,1]\to\mathbb{R}_0$ *and* $f:I\subseteq\mathbb{R}_+\to\mathbb{R}_+$. *Then* $f\in\mathrm{HGX}(h,I)$ *if and only if* $\frac{1}{f}\in\mathrm{HGV}(h,I)$.

Proof. If $f\in\mathrm{HGX}(h,I)$, we have

$$f\left(x^t y^{1-t}\right)\leqslant[f(x)]^{h(t)}[f(y)]^{h(1-t)}$$

for $x,y\in I$ and $t\in[0,1]$, so

$$\left[f\left(x^t y^{1-t}\right)\right]^{-1}\geqslant[f(x)]^{-h(t)}[f(y)]^{-h(1-t)}$$

for all $x,y\in I$ and $t\in[0,1]$, namely, $\frac{1}{f}\in\mathrm{HGV}(h,I)$.

Similarly, if $\frac{1}{f}\in\mathrm{HGV}(h,I)$, then $f\in\mathrm{HGX}(h,I)$. Theorem 3.2 is proved. □

THEOREM 3.3. *Let* $h:[0,1]\to\mathbb{R}_0$, $f:I\subseteq\mathbb{R}_+\to\mathbb{R}_+$, *and* $\varphi:J\subseteq\mathbb{R}_+\to\varphi(J)\subseteq I$.

1. *If* $f\in\mathrm{HGX}(h,I)$ *is an increasing* (*or decreasing respectively*) *function on* I *and* φ *is geometrically convex* (*or concave respectively*) *function on* J, *then* $f\circ\varphi\in\mathrm{HGX}(h,J)$.
2. *If* $f\in\mathrm{HGV}(h,I)$ *is an increasing* (*or decreasing respectively*) *function on* I *and* φ *is geometrically convex* (*or concave respectively*) *function on* J, *then* $f\circ\varphi\in\mathrm{HGV}(h,J)$.

Proof. We only prove the case that $f\in\mathrm{HGX}(h,I)$ is an decreasing function on I and φ is a geometrically concave function on J.

Since φ is a geometrically concave function on J, for all $x,y\in J$ and $t\in[0,1]$, we have

$$\varphi\left(x^t y^{1-t}\right)\geqslant[\varphi(x)]^t[\varphi(y)]^{1-t}.$$

Since f is an decreasing and h-geometrically convex function on I, we have

$$f\left(\varphi\left(x^t y^{1-t}\right)\right)\leqslant f\left([\varphi(x)]^t[\varphi(y)]^{1-t}\right)\leqslant[f(\varphi(x))]^{h(t)}[f(\varphi(y))]^{h(1-t)}$$

for all $x,y\in J$ and $t\in[0,1]$, and then $f\circ\varphi\in\mathrm{HGX}(h,J)$. Theorem 3.3 is thus proved. □

THEOREM 3.4. *Let* $h_i : [0,1] \to \mathbb{R}_0$ *for* $i = 1,2$ *and* $f : I \subseteq \mathbb{R}_+ \to \mathbb{R}_+$. *If*

1. *either* $h_2(t) \geqslant h_1(t)$ *for* $t \in [0,1]$, $f : I \subseteq \mathbb{R}_+ \to (0,1]$, *and* $f \in \mathrm{HGX}(h_2, I)$,
2. *or* $h_2(t) \leqslant h_1(t)$ *for* $t \in [0,1]$, $f : I \subseteq \mathbb{R}_+ \to [1,\infty)$, *and* $f \in \mathrm{HGX}(h_2, I)$,

then $f \in \mathrm{HGX}(h_1, I)$.

Proof. This is an easy consequence of Definition 2.1. □

COROLLARY 3.1. *Let* $h_i : [0,1] \to \mathbb{R}_0$ *for* $i = 1,2,\dots,n$ *and* $f : I \subseteq \mathbb{R}_+ \to \mathbb{R}_+$. *If*

1. *either* $h(t) = \min\{h_i(t), 1 \leqslant i \leqslant n\}$ *for* $t \in [0,1]$ *and* $f : I \subseteq \mathbb{R}_+ \to (0,1]$,
2. *or* $h(t) = \max\{h_i(t), 1 \leqslant i \leqslant n\}$ *for* $t \in [0,1]$ *and* $f : I \subseteq \mathbb{R}_+ \to [1,\infty)$,

then $f \in \mathrm{HGX}(h, I)$.

Proof. This follows from utilizing Theorem 3.4 and induction. □

THEOREM 3.5. *Let* $h : [0,1] \to \mathbb{R}_0$ *and* $f \in \mathrm{HGX}(h, I)$.

1. *If* $h(t) + h(1-t) \geqslant 1$ *for* $t \in [0,1]$, *then* $f(x) \geqslant 1$ *for* $x \in I$;
2. *If* $h(t) + h(1-t) \leqslant 1$ *for* $t \in [0,1]$, *then* $f(x) \leqslant 1$ *for* $x \in I$.

Proof. Since $f \in \mathrm{HGX}(h, I)$, we have

$$f(x) = f\left(x^t x^{1-t}\right) \leqslant [f(x)]^{h(t)}[f(x)]^{h(1-t)} = [f(x)]^{h(t)+h(1-t)},$$

which can be rearranged as

$$[f(x)]^{h(t)+h(1-t)-1} \geqslant 1.$$

By the property of the exponential function, Theorem 3.5 follows. □

4. Inequalities

THEOREM 4.1. *Let* $h : [0,1] \to \mathbb{R}_0$ *and* $f \in \mathrm{HGX}(h, I)$ *such that* $1 \in I$. *If*

1. *either* h *is a sub-multiplicative function on* $[0,1]$ *and* $f : I \to (0,1]$,
2. *or* h *is a super-multiplicative function on* $[0,1]$ *and* $f : I \to [1,\infty)$,

then, for all $\alpha, \beta > 0$ *and* $x, y \in I$, *with* $\alpha + \beta = \gamma \leqslant 1$, *we have*

$$f\left(x^\alpha y^\beta\right) \leqslant [f(x)]^{h(\alpha)}[f(y)]^{h(\beta)}[f(1)]^{h(\alpha(1-\gamma)/\gamma)+h(\beta(1-\gamma)/\gamma)}. \tag{4.1}$$

Specially, if $f(1) = 1$, *then*

$$f\left(x^\alpha y^\beta\right) \leqslant [f(x)]^{h(\alpha)}[f(y)]^{h(\beta)}. \tag{4.2}$$

Proof. If $h:[0,1]\to\mathbb{R}_0$ is a sub-multiplicative function and $f:I\to(0,1]$, let $\lambda=\frac{\alpha}{\gamma}$, then

$$\begin{aligned}
f\left(x^{\alpha}y^{\beta}\right)&=f\left(x^{\lambda\gamma}y^{(1-\lambda)\gamma}\right)\leqslant[f(x^{\gamma})]^{h(\lambda)}[f(y^{\gamma})]^{h(1-\lambda)}\\
&\leqslant\{[f(x)]^{h(\gamma)}[f(1)]^{h(1-\gamma)}\}^{h(\lambda)}\{[f(y)]^{h(\gamma)}[f(1)]^{h(1-\gamma)}\}^{h(1-\lambda)}\\
&\leqslant[f(x)]^{h(\lambda\gamma)}[f(y)]^{h((1-\lambda)\gamma)}[f(1)]^{h((\lambda\gamma)+h((1-\lambda)(1-\gamma))}\\
&=[f(x)]^{h(\alpha)}[f(y)]^{h(\beta)}[f(1)]^{h(\alpha(1-\gamma)/\gamma)+h(\beta(1-\gamma)/\gamma)}
\end{aligned}$$

for $x,y\in I$. Specially, if $f(1)=1$, we can obtain the inequality (4.2) easily. The proof of Theorem 4.1 is complete. □

THEOREM 4.2. *Let $h:[0,1]\to\mathbb{R}_0$, $f\in \mathrm{HGX}(h,I)$, and $\lambda_i>0$ with $\sum_{i=1}^{n}\lambda_i=1$. If*

1. *either h is a sub-multiplicative function on $[0,1]$ and $f:I\to(0,1]$,*
2. *or h is a super-multiplicative function on $[0,1]$ and $f:I\to[1,\infty)$,*

then, for all $x_i\in I$ and $i=1,2,\dots,n$, we have

$$f\left(\prod_{i=1}^{n}x_i^{\lambda_i}\right)\leqslant\prod_{i=1}^{n}[f(x_i)]^{h(\lambda_i)}. \tag{4.3}$$

Proof. When $n=2$, from Definition 2.1, we have

$$f\left(x_1^{\lambda_1}x_2^{\lambda_2}\right)\leqslant[f(x_1)]^{h(\lambda_1)}[f(x_2)]^{h(\lambda_2)}$$

for $x_1,x_2\in I$. So the inequality (4.3) holds for $n=2$.

Suppose that the inequality (4.3) holds for $n=k$, i.e.,

$$f\left(\prod_{i=1}^{k}x_i^{\lambda_i}\right)\leqslant\prod_{i=1}^{k}[f(x_i)]^{h(\lambda_i)}$$

for $x_i\in I$ and $i=1,2,\dots,k$. By this hypothesis, it follows that, when $n=k+1$, putting $\Lambda_k=\sum_{i=1}^{k}\lambda_i$ gives

$$\begin{aligned}
f\left(\prod_{i=1}^{k+1}x_i^{\lambda_i}\right)&=f\left(x_{k+1}^{\lambda_{k+1}}\left(\prod_{i=1}^{k}x_i^{\lambda_i}\right)\right)=f\left(x_{k+1}^{\lambda_{k+1}}\left(\prod_{i=1}^{k}x_i^{\lambda_i/\Lambda_k}\right)^{\Lambda_k}\right)\\
&\leqslant[f(x_{k+1})]^{h(\lambda_{k+1})}\left[f\left(\prod_{i=1}^{k}x_i^{\lambda_i/\Lambda_k}\right)\right]^{h(\Lambda_k)}=[f(x_{k+1})]^{h(\lambda_{k+1})}\prod_{i=1}^{k}[f(x_i)]^{h(\lambda_i/\Lambda_k)h(\Lambda_k)}.
\end{aligned}$$

Since h is a sub-multiplicative function on $[0,1]$ and $f:I\to(0,1]$, or h is a super-multiplicative function on $[0,1]$ and $f:I\to[1,\infty)$, we have

$$f\left(\prod_{i=1}^{k+1}x_i^{\lambda_i}\right)\leqslant[f(x_{k+1})]^{h(\lambda_{k+1})}\prod_{i=1}^{k}[f(x_i)]^{h(\lambda_i/\Lambda_k)h(\Lambda_k)}\leqslant\prod_{i=1}^{k+1}[f(x_i)]^{h(\lambda_i)}$$

for $x_i \in I$ and $i = 1,2,\ldots,n$. Equivalently speaking, when $n = k+1$, the inequality (4.3) holds. By induction, the proof of Theorem 4.2 is complete. □

THEOREM 4.3. *Let $h:[0,1] \to \mathbb{R}_0$ and $f : I \subseteq \mathbb{R}_+ \to \mathbb{R}_+$ satisfying $f \in \mathrm{HGX}(h_1, [a,b])$ and $f \in L([a,b])$, where $a,b \in I$ with $0<a<b$. If*

1. *either $h(t) \geqslant t$ for $t \in [0,1]$ and $f : [a,b] \to (0,1]$,*
2. *or $h(t) \leqslant t$ for $t \in [0,1]$ and $f : [a,b] \to [1,\infty)$,*

then

$$\frac{1}{\ln b - \ln a}\int_a^b \frac{f(x)}{x}\,\mathrm{d}x \leqslant L(f(a),f(b)) \tag{4.4}$$

and

$$\frac{1}{\ln b - \ln a}\int_a^b f(x)\,\mathrm{d}x \leqslant L(af(a),bf(b)), \tag{4.5}$$

where $L(u,v)$ is the logarithmic mean defined by

$$L(u,v) = \begin{cases} \dfrac{u-v}{\ln u - \ln v}, & u \neq v; \\ u, & u = v. \end{cases} \tag{4.6}$$

Proof. Let $x = a^{1-t}b^t$ for $0 \leqslant t \leqslant 1$. Using the h-geometric convexity of f on $[a,b]$, either utilizing $h(t) \geqslant t$ for $t \in [0,1]$ and $f : I \subseteq \mathbb{R}_+ \to (0,1]$, or employing $h(t) \leqslant t$ for $t \in [0,1]$ and $f : I \subseteq \mathbb{R}_+ \to [1,\infty)$, we obtain

$$\begin{aligned}\frac{1}{\ln b - \ln a}\int_a^b \frac{f(x)}{x}\,\mathrm{d}x &= \int_0^1 f\big(a^{1-t}b^t\big)\,\mathrm{d}t \leqslant \int_0^1 [f(a)]^{h(1-t)}[f(b)]^{h(t)}\,\mathrm{d}t \\ &\leqslant \int_0^1 [f(a)]^{1-t}[f(b)]^t\,\mathrm{d}t = L(f(a),f(b))\end{aligned}$$

and

$$\begin{aligned}\frac{1}{\ln b - \ln a}\int_a^b f(x)\,\mathrm{d}x &= \int_0^1 a^{1-t}b^t f\big(a^{1-t}b^t\big)\,\mathrm{d}t \leqslant \int_0^1 a^{1-t}b^t[f(a)]^{h(1-t)}[f(b)]^{h(t)}\,\mathrm{d}t \\ &\leqslant \int_0^1 a^{1-t}b^t[f(a)]^{1-t}[f(b)]^t\,\mathrm{d}t = L(af(a),bf(b)).\end{aligned}$$

The proof of Theorem 4.3 is complete. □

COROLLARY 4.1. *Under conditions of Theorem 4.3, if $h(t) = t$, then*

$$\frac{1}{\ln b - \ln a}\int_a^b \frac{f(x)}{x}\,\mathrm{d}x \leqslant L(f(a),f(b)) \tag{4.7}$$

and

$$\frac{1}{\ln b - \ln a}\int_a^b f(x)\,\mathrm{d}x \leqslant L(af(a),bf(b)). \tag{4.8}$$

THEOREM 4.4. *Let* $h_i : [0,1] \to \mathbb{R}_0$ *for* $i = 1,2$ *and* $f,g : I \subseteq \mathbb{R}_+ \to \mathbb{R}_+$ *satisfying* $f \in \mathrm{HGX}(h_1,[a,b])$, $g \in \mathrm{HGX}(h_2,[a,b])$, *and* $fg \in L([a,b])$, *where* $a,b \in I$ *with* $0 < a < b$. *If*

1. *either* $\min\{h_1(t),h_2(t)\} \geqslant t$ *for* $t \in [0,1]$ *and* $f,g : [a,b] \to (0,1]$,
2. *or* $\max\{h_1(t),h_2(t)\} \leqslant t$ *for* $t \in [0,1]$ *and* $f,g : [a,b] \to [1,\infty)$,

then

$$\frac{1}{\ln b - \ln a}\int_a^b \frac{f(x)}{x} g(x)\,\mathrm{d}x \leqslant L(f(a)g(a), f(b)g(b)) \tag{4.9}$$

and

$$\frac{1}{\ln b - \ln a}\int_a^b f(x)g(x)\,\mathrm{d}x \leqslant L(af(a)g(a), bf(b)g(b)), \tag{4.10}$$

where $L(u,v)$ *is defined as in* (4.6).

COROLLARY 4.2. *Under conditions of Theorem* 4.4, *if* $h(t) = t$, *then*

$$\frac{1}{\ln b - \ln a}\int_a^b \frac{f(x)}{x} g(x)\,\mathrm{d}x \leqslant L(f(a)g(a), f(b)g(b)) \tag{4.11}$$

and

$$\frac{1}{\ln b - \ln a}\int_a^b f(x)g(x)\,\mathrm{d}x \leqslant L(af(a)g(a), bf(b)g(b)). \tag{4.12}$$

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