Journal of
Classical
Analysis

Volume 3, Number 2 (2013), 157–165

doi:10.7153/jca-03-14

CONSTRUCTION MODELS OF GAUSSIAN RANDOM PROCESSES WITH A GIVEN ACCURACY AND RELIABILITY IN $L_p(T)$, $p \geqslant 1$

NATALIYA TROSHKI

Abstract. The model of some Gaussian random process which approximates it with a given accuracy and reliability in $L_p(T), p \geqslant 1$ is constructed in this paper. The sub-Gaussian theory of random variables is used for finding the model with given accuracy and reliability.

1. Introduction

Modelling of stochastic processes has been extensively studied in the recent years. Ermakov and Mykhaylov considered the basic principles of the construction models for Gaussian stochastic processes in the book [3]. Sabel'fel'd and Kurbanmuradov studied some general methods of modelling the Gaussian but not only the stationary stochastic processes in [9]. Issues of accuracy and reliability of modelling the stationary Gaussian stochastic processes in $L_p(T)$ have been discussed in the papers by Antonini, Kozachenko and Tegza [1] and Kozachenko and Rozora [8].

However, if the process is not stationary, the questions about the accuracy and reliability of the constructed models has not yet been fully investigated. Recently, a considerable attention was also given to wavelet series representations of stochastic processes in $L_p([0,T])$, see, for example [7].

The main objective of the work is to construct the model of Gaussian stochastic process which approximates it in $L_p(T)$, $p \geqslant 1$ with a given accuracy and reliability. We focus on the model for Gaussian real zero-mean stochastic process with the following covariance function

$$R(t,s) = \int_0^\infty g(t,\lambda)g(s,\lambda)dF(\lambda).$$

It was proved that the approximating process discussed in this paper is the process close to Gaussian, namely sub-Gaussian stochastic process. Sub-Gaussian random variables appeared in paper by Kahane [4]. More information on the theory of sub-Gaussian random variables and stochastic processes can be found in the books [2, 8].

The organization of the article is the following. The model of Gaussian random process is constructed in the Section 2. Our main result (Theorem 3.2) is proved in the Section 3. Section 4 is devoted to conclusions.

Mathematics subject classification (2010): Primary 41A25; secondary 60F10.
Keywords and phrases: Gaussian random processes, accuracy and reliability, modelling, sub-Gaussian random variables.

Paper JCA-03-14

2. Constructing a model of Gaussian random process

Let $\{\Omega, \mathscr{B}, \mathscr{P}\}$ be a usual, fixed probability space, $T = [0, A]$ be a parametric set. Let $\xi = \{\xi(t), t \in T\}$ be a zero-mean real-valued Gaussian stochastic process. The covariance function of the process is defined as

$$R(t,s) = \int_0^\infty g(t,\lambda)g(s,\lambda)dF(\lambda),$$

where $F(\lambda)$ is a distribution function.

According to the Karhunen theorem [5], the process ξ can be represented as follows

$$\xi(t) = \int_0^\infty g(t,\lambda)d\eta(\lambda),$$

where $\eta(\lambda)$ is a Gaussian process with independent increments, that is $\mathbb{E}(\eta(b) - \eta(c))^2 = F(b) - F(c)$, $b > c$, and $\mathbb{E}\eta(\lambda) = 0$.

Let the following condition hold for the function $g(t,\lambda)$

$$| g(t,\lambda) - g(t,u) | \leqslant S(| u - \lambda |) \cdot Z(t), \tag{1}$$

where $Z(t)$ is some continuous function and $S(\lambda), \lambda \in \mathbb{R}$ monotone increases.

Let $L > 0$ be a given real number. We consider such partition $\Lambda = \{\lambda_0, ..., \lambda_N\}$ of the set $[0, \infty]$ that $\lambda_0 = 0, \lambda_k < \lambda_{k+1}, \lambda_{N-1} = L, \lambda_N = +\infty$. For this partition we can write

$$\xi(t) = \sum_{k=0}^{N-1} \int_{\lambda_k}^{\lambda_{k+1}} g(t,\lambda)d\eta(\lambda).$$

As a model for the process ξ we consider

$$\xi_\Lambda(t) = \sum_{k=0}^{N-1} \eta_k g(t, \zeta_k),$$

where η_k and ζ_k are independent random variables, η_k are such Gaussian random variables that $\mathbb{E}\eta_k = 0$, $\mathbb{E}\eta_k^2 = F(\lambda_{k+1}) - F(\lambda_k) = b_k^2$; ζ_k are random variables taking values on the segments $[\lambda_k, \lambda_{k+1}]$ and if $b_k^2 > 0$, then

$$F_k(\lambda) = P\{\zeta_k < \lambda\} = \frac{F(\lambda) - F(\lambda_k)}{F(\lambda_{k+1}) - F(\lambda_k)}.$$

If $b_k^2 = 0$, then $\zeta_k = 0$ with probability one. For the sake of simplicity we assume that $b_k^2 > 0$, $k = 0, 1, ..., N$.

This model is a zero-mean process

$$\mathbb{E}\xi_\Lambda(t) = \mathbb{E}\sum_{k=0}^{N-1} \eta_k g(t, \zeta_k) = \sum_{k=0}^{N-1} \mathbb{E}\eta_k \mathbb{E}g(t, \zeta_k) = 0.$$

The covariance function of the process $\xi_\Lambda(t)$ coincides with covariance function of $\xi(t)$

$$\mathbb{E}\xi_\Lambda(t)\xi_\Lambda(s)=\mathbb{E}\left(\sum_{k=0}^{N-1}\eta_k g(t,\zeta_k)\right)\left(\sum_{k=0}^{N-1}\eta_k g(s,\zeta_k)\right)=\sum_{k=0}^{N-1}\mathbb{E}\eta_k^2\cdot\mathbb{E}g(t,\zeta_k)g(s,\zeta_k)$$
$$=\sum_{k=0}^{N-1}b_k^2\int_{\lambda_k}^{\lambda_{k+1}}g(t,\lambda)g(s,\lambda)dF_k(\lambda)=\int_0^\infty g(t,\lambda)g(s,\lambda)dF(\lambda).$$

Putting $\eta_k=\int_{\lambda_k}^{\lambda_{k+1}}d\eta(\lambda)$ we consider the following difference

$$\begin{aligned}\xi(t)-\xi_\Lambda(t)&=\sum_{k=0}^{N-1}\int_{\lambda_k}^{\lambda_{k+1}}g(t,\lambda)d\eta(\lambda)-\sum_{k=0}^{N-1}\int_{\lambda_k}^{\lambda_{k+1}}g(t,\zeta_k)d\eta(\lambda)\\&=\sum_{k=0}^{N-1}\int_{\lambda_k}^{\lambda_{k+1}}(g(t,\lambda)-g(t,\zeta_k))\,d\eta(\lambda).\end{aligned}\tag{2}$$

LEMMA 2.1. *Let condition* (1) *holds for a function* $g(t,\lambda)$. *The we have*

$$\mathbb{E}\left(\int_{\lambda_k}^{\lambda_{k+1}}(g(t,\lambda)-g(t,\zeta_k))\,d\eta(\lambda)\right)^{2m+1}=0,$$

$$\mathbb{E}\left(\int_{\lambda_k}^{\lambda_{k+1}}(g(t,\lambda)-g(t,\zeta_k))\,d\eta(\lambda)\right)^{2m}\leqslant\frac{(2m)!}{2^m\cdot m!}Z^{2m}(t)\mathbb{E}\left(\int_{\lambda_k}^{\lambda_{k+1}}S^2(|\lambda-\zeta_k|)dF(\lambda)\right)^m.$$

Proof. Since for a zero-mean Gaussian random variable ξ it is $\mathbb{E}\xi=0$, $\mathbb{E}\xi^{2k+1}=0$, $\mathbb{E}\xi^{2k}=\frac{(2k)!}{2^k\cdot k!}\sigma^{2k}$ and the random variables ζ_k are independent of $\eta(\lambda)$, then by the Fubini's theorem ($\mathbb{E}_{\zeta_k}$ is a conditional expectation with respect to ζ_k):

$$\mathbb{E}\left(\int_{\lambda_k}^{\lambda_{k+1}}(g(t,\lambda)-g(t,\zeta_k))\,d\eta(\lambda)\right)^{2m}$$
$$=\mathbb{E}\mathbb{E}_{\zeta_k}\left(\int_{\lambda_k}^{\lambda_{k+1}}(g(t,\lambda)-g(t,\zeta_k))\,d\eta(\lambda)\right)^{2m}=\frac{(2m)!}{2^m\cdot m!}\mathbb{E}\left(\int_{\lambda_k}^{\lambda_{k+1}}|\,g(t,\lambda)-g(t,\zeta_k)\,|^2\,dF(\lambda)\right)^m$$
$$\leqslant\frac{(2m)!}{2^m\cdot m!}\mathbb{E}\left(\int_{\lambda_k}^{\lambda_{k+1}}S^2(|\,\lambda-\zeta_k\,|)Z^2(t)dF(\lambda)\right)^m=\frac{(2m)!}{2^m\cdot m!}Z^{2m}(t)\mathbb{E}\left(\int_{\lambda_k}^{\lambda_{k+1}}S^2(|\,\lambda-\zeta_k\,|)dF(\lambda)\right)^m,$$

which finishes the proof. □

DEFINITION 2.1. [2] A random variable ξ is sub-Gaussian if there exists $a \geqslant 0$, such that the inequality

$$\mathbb{E}\exp\{\lambda\xi\} \leqslant \exp\left\{\frac{a^2\lambda^2}{2}\right\}$$

holds for all $\lambda \in \mathbb{R}$.

The space of all sub-Gaussian random variables defined on a common probability space $\{\Omega, \mathscr{B}, \mathscr{P}\}$ we denote $Sub(\Omega)$. The space $Sub(\Omega)$ is a Banach space with respect to the norm

$$\tau(\xi) = \sup_{\lambda \neq 0}\left[\frac{2\ln \mathbb{E}\exp\{\lambda\xi\}}{\lambda^2}\right]^{\frac{1}{2}}.$$

DEFINITION 2.2. [2] Let T be a parametric set. A stochastic process $X = \{X(t), t \in T\}$ is called sub-Gaussian if for all $t \in T$, $X(t) \in Sub(\Omega)$ and $\sup_{t\in T} \tau(X(t)) < \infty$.

LEMMA 2.2. [2] *Let ξ be a zero-mean random variable such that $\mathbb{E}\xi^{2k+1} = 0$ and $\theta(\xi) = \sup_{k\geqslant 1}\left[\frac{2^k k!}{(2k)!}\mathbb{E}\xi^{2k}\right]^{\frac{1}{2k}} < \infty$. Then $\xi \in Sub(\Omega)$ and $\tau(\xi) \leqslant \theta(\xi)$.*

THEOREM 2.1. *The stochastic process $\xi(t) - \xi_\Lambda(t)$ is sub-Gaussian and the following inequality holds*

$$\tau(\xi(t) - \xi_\Lambda(t)) \leqslant Z(t)\left[\sum_{k=0}^{N-1} b_k^2 \sup_{m\geqslant 1}\left(\mathbb{E}S^{2m}(|\,\zeta_k - \zeta_k^*\,|)\right)^{\frac{1}{m}}\right]^{\frac{1}{2}},$$

where $b_k^2 = F(\lambda_{k+1}) - F(\lambda_k)$ and ζ_k^* are random variables independent of ζ_k but with the same distribution as ζ_k.

Proof. Using the Lemma 2.1 we obtain

$$\tau^2\left(\int_{\lambda_k}^{\lambda_{k+1}} (g(t,\lambda) - g(t,\zeta_k))d\eta(\lambda)\right)$$

$$\leqslant \theta^2\left(\int_{\lambda_k}^{\lambda_{k+1}} (g(t,\lambda) - g(t,\zeta_k))d\eta(\lambda)\right)$$

$$\leqslant \sup_{m\geqslant 1} b_k^2 Z^2(t)\left(\mathbb{E}\left(\int_{\lambda_k}^{\lambda_{k+1}} S^2(|\,\lambda - \zeta_k\,|)dF_k(\lambda)\right)^m\right)^{\frac{1}{m}}$$

$$= \sup_{m\geqslant 1} b_k^2 Z^2(t)\left(\int_{\lambda_k}^{\lambda_{k+1}}\left(\int_{\lambda_k}^{\lambda_{k+1}} S^2(|\,\lambda - u\,|)dF_k(\lambda)\right)^m dF_k(u)\right)^{\frac{1}{m}},$$

Lemma 2.2 implies that $\int\limits_{\lambda_k}^{\lambda_{k+1}} (g(t,\lambda)-g(t,\zeta_k))d\eta(\lambda)$ are sub-Gaussian random variables.

Since the terms in the sum (2) for different k are independent, so from the last equality we have

$$\tau^2(\xi(t)-\xi_\Lambda(t)) \leqslant Z^2(t)\sum_{k=0}^{N-1} b_k^2 \sup_{m\geqslant 1}\left(\int\limits_{\lambda_k}^{\lambda_{k+1}}\left(\int\limits_{\lambda_k}^{\lambda_{k+1}} S^2(|\,\lambda-u\,|)dF_k(\lambda)\right)^m dF_k(u)\right)^{\frac{1}{m}}.$$

Then, from the Fubini's theorem and the Lyapunov inequality we obtain

$$\begin{aligned}
\tau(\xi(t)-\xi_\Lambda(t)) &\leqslant Z(t)\left[\sum_{k=0}^{N-1}\sup_{m\geqslant 1} b_k^2\left(\int\limits_{\lambda_k}^{\lambda_{k+1}}\left(\int\limits_{\lambda_k}^{\lambda_{k+1}} S^2(|\lambda-u|)dF_k(\lambda)\right)^m dF_k(u)\right)^{\frac{1}{m}}\right]^{\frac{1}{2}}\\
&= Z(t)\left[\sum_{k=0}^{N-1}\sup_{m\geqslant 1} b_k^2\left(\mathbb{E}_{\zeta_k^*}\left(\mathbb{E}_{\zeta_k}S^2(|\zeta_k-\zeta_k^*|)\right)^m\right)^{\frac{1}{m}}\right]^{\frac{1}{2}}\\
&\leqslant Z(t)\left[\sum_{k=0}^{N-1}\sup_{m\geqslant 1} b_k^2\left(\mathbb{E}_{\zeta_k^*}\mathbb{E}_{\zeta_k}S^{2m}(|\zeta_k-\zeta_k^*|)\right)^{\frac{1}{m}}\right]^{\frac{1}{2}}\\
&\leqslant Z(t)\left[\sum_{k=0}^{N-1} b_k^2\sup_{m\geqslant 1}\left(\mathbb{E}S^{2m}(|\zeta_k-\zeta_k^*|)\right)^{\frac{1}{m}}\right]^{\frac{1}{2}},
\end{aligned}$$

which is the desired statement. □

COROLLARY 2.1. *If for all $\lambda,u\in\mathbb{R}_+$ there exists an absolute constant $C>0$ so, that*

$$|\,g(t,\lambda)-g(t,u)\,|\leqslant C$$

then we have

$$\tau(\xi(t)-\xi_\Lambda(t))\leqslant Z(t)\left[\sum_{k=0}^{N-2} b_k^2\sup_{m\geqslant 1}\left(\mathbb{E}S^{2m}(|\,\zeta_k-\zeta_k^*\,|)\right)^{\frac{1}{m}}+C^2(F(+\infty)-F(\Lambda))\right]^{\frac{1}{2}},$$

where b_k,ζ_k^* remain the same as in the previous Theorem 2.1.

EXAMPLE 2.1. Let covariance function of stochastic process ξ have the following form

$$R(t,s)=\int\limits_0^\infty \cos(t\lambda)\cos(s\lambda)dF(\lambda),$$

i.e. $g(t,\lambda)=\cos(t\lambda)$. Then $\xi(t)=\int\limits_0^\infty \cos(t\lambda)d\eta(\lambda)$ is a zero-mean real-valued Gaussian stochastic process, where $\eta(\lambda)$ is a Gaussian process with independent increments, $\mathbb{E}(\eta(b)-\eta(c))^2=F(b)-F(c), b>c$.

Consider the following straightforward estimate:

$$\cos(t\lambda)-\cos(tu)|^2 = |2\sin\frac{t(\lambda-u)}{2}\sin\frac{t(\lambda+u)}{2}|^2$$
$$\leqslant |2\sin\frac{t(u-\lambda)}{2}|^2 \leqslant 2^{2(1-\alpha)}t^{2\alpha}|u-\lambda|^{2\alpha}, \qquad 0<\alpha\leqslant 1,$$

By virtue of Theorem 2.1. and 2.1 and taking into account that the functions $Z(t)=2^{(1-\alpha)}t^\alpha$, $S(\lambda)=\lambda^\alpha$, while $C=2$ we obtain the following inequality

$$\tau^2(\xi(t)-\xi_\Lambda(t)) \leqslant 2^{2(1-\alpha)}t^{2\alpha}\sum_{k=0}^{N-2} b_k^2 \mid \lambda_{k+1}-\lambda_k\mid^{2\alpha} + 4(F(+\infty)-F(\Lambda)).$$

EXAMPLE 2.2. Consider the covariance function of stochastic process ξ which have the following form

$$R(t,s)=\mathbb{E}\xi(t)\xi(s)=\int\limits_0^\infty J_l(t,\lambda)J_l(s,\lambda)dF(\lambda),$$

i.e. $g(t,\lambda)=J_l(t,\lambda)$, where $J_l(t,\lambda)=\frac{1}{\pi}\int\limits_0^\pi \cos(l\varphi-t\lambda\sin\varphi)d\varphi$.

Then $\xi(t)=\int\limits_0^\infty J_l(t,\lambda)d\eta(\lambda)$ is a zero-mean real-valued Gaussian stochastic process, where $\eta(\lambda)$ is a Gaussian process with independent increments, $\mathbb{E}(\eta(b)-\eta(c))^2 = F(b)-F(c)$, $b>c$, $\mathbb{E}\eta(\lambda)=0$.

Let us find the estimate for the squared difference $\Delta_J(\lambda,u)=|J_l(t,\lambda)-J_l(t,u)|^2$. By direct calculations we get

$$\Delta_J(\lambda,u)=\frac{1}{\pi^2}\left|\int\limits_0^\pi 2\sin\left(\frac{2l\varphi-t(\lambda+u)\sin\varphi}{2}\right)\sin\left(\frac{t(u-\lambda)\sin\varphi}{2}\right)d\varphi\right|^2$$
$$\leqslant \frac{1}{\pi^2}\int\limits_0^\pi |2\sin\frac{t(u-\lambda)\sin\varphi}{2}|^2 d\varphi \leqslant \frac{1}{\pi^2}\int\limits_0^\pi t^2|u-\lambda|^2\sin^2\varphi d\varphi = \frac{t^2}{\pi}|u-\lambda|^2.$$

Applying Theorem 2.1 and Corollary 2.1 and having in mind that $Z(t)=t/\sqrt{\pi}$, $S(\lambda)=\lambda$ and $C=1$, we arrive at the following inequality

$$\tau^2(\xi(t)-\xi_\Lambda(t)) \leqslant \frac{t^2}{\pi}\sum_{k=0}^{N-2} b_k^2|\lambda_{k+1}-\lambda_k|^2+(F(+\infty)-F(\Lambda)).$$

3. Accuracy and reliability the model for Gaussian stochastic process in space $L_p(T)$, $p \geqslant 1$.

DEFINITION 3.1. [8] A stochastic process $\xi_\Lambda(t)$ approximates the process $\xi(t)$ with accuracy $\varepsilon > 0$ and reliability $(1-\delta)$, $0 < \delta < 1$ in $L_p(T)$, if the partition Λ is such, that the following inequality holds

$$\mathbf{P}\left\{\left(\int_T |\,\xi(t)-\xi_\Lambda(t)\,|^p\,dt\right)^{\frac{1}{p}} > \varepsilon\right\} \leqslant \delta.$$

THEOREM 3.1. [6] *Suppose that* $\xi = \{\xi(t), t \in T\}$ *is a sub-Gaussian stochastic process,* $\mathbb{E}\xi(t) = 0$, $\tau^2(t) = \tau^2(\xi(t)) = \mathbb{E}(\xi(t))^2$. *Suppose there exists an integral* $\int_T \left(\mathbb{E}(\xi(t))^2\right)^{\frac{p}{2}} dt < \infty$, $p \geqslant 1$. *Then the integral* $\int_T |\,\xi(t)\,|^p\,dt < \infty$, *exists with probability* 1 *and for all* ε *satisfying* $\varepsilon > c_p^{1/p}\sqrt{p}$, *where* $c_p = \int_T (\tau(t))^p\,dt$, *we have*

$$\mathbf{P}\{\|\,\xi(t)\,\|_{L_p} > \varepsilon\} \leqslant 2\exp\left\{-\frac{\varepsilon^2}{2c_p^{2/p}}\right\}.$$

THEOREM 3.2. *Suppose that the partition* Λ *in the model* $\xi_\Lambda(t)$ *is such, that*

$$\int_T (\tau(\xi(t)-\xi_\Lambda(t)))^p\,dt \leqslant \frac{\varepsilon^p}{\max\left(p^{p/2}, \left(2\ln\frac{2}{\delta}\right)^{p/2}\right)}.$$

Then this model approximates the Gaussian process $\xi(t)$ *with accuracy* $\varepsilon > 0$ *and reliability* $1-\delta$, $0 < \delta < 1$ *in the space* $L_p(T)$, $p \geqslant 1$.

Proof. If $\varepsilon > \sqrt{p}\,c_p^{1/p}$, then according to Theorem 3.1 and Definition 3.1 we have

$$\mathbf{P}\{\|\,\xi(t)-\xi_\Lambda(t)\,\|_{L_p} > \varepsilon\} \leqslant 2\exp\left\{-\frac{\varepsilon^2}{2c_p^{2/p}}\right\} \leqslant \delta.$$

Accordingly, the last estimate is valid when

$$\int_T (\tau(\xi(t)-\xi_\Lambda(t)))^p\,dt \leqslant \frac{\varepsilon^p}{\left(2\ln\frac{2}{\delta}\right)^{\frac{p}{2}}}.$$

The proof is completed. □

EXAMPLE 3.1. Let $F(\lambda)$ in Example 2.1 be such that $F(+\infty) = 1$, $F(+\infty) - F(L) \leqslant L^{-\alpha}$, $0 \leqslant \alpha \leqslant 1$, $L > 1$. With the aid of the Corollary 2.1 we get

$$\tau^2(\xi(t)-\xi_\Lambda(t)) \leqslant 2^{2(1-\alpha)}t^{2\alpha}\sum_{k=0}^{N-2} b_k^2\,|\,\lambda_{k+1}-\lambda_k\,|^{2\alpha} + 4(F(+\infty)-F(L)).$$

Letting $|\lambda_{k+1}-\lambda_k| = L/N$ we conclude

$$\tau^2(\xi(t)-\xi_\Lambda(t)) \leqslant 2^{2(1-\alpha)}t^{2\alpha}F(L)\left(\frac{L}{N}\right)^{2\alpha} + 4(F(+\infty)-F(L)) \leqslant 4\left(\frac{tL}{2N}\right)^{2\alpha} + \frac{4}{L^\alpha},$$

hence

$$\tau(\xi(t)-\xi_\Lambda(t)) \leqslant 2\left[\left(\frac{tL}{2N}\right)^{2\alpha} + \frac{1}{L^\alpha}\right]^{\frac{1}{2}}.$$

Next, minimize $y_1(L) = 2\left[\left(\frac{tL}{2N}\right)^{2\alpha} + \frac{1}{L^\alpha}\right]^{1/2}$ with respect to L; it follows that

$$y(L_0) = \min, \qquad L_0 = \frac{1}{4^{1/(6\alpha)}}\left(\frac{2N}{t}\right)^{2/3}.$$

Then

$$\tau(\xi(t)-\xi_\Lambda(t)) \leqslant 2\left[\left(\frac{t}{2N}\right)^{2\alpha}\frac{1}{\sqrt[3]{4}}\left(\frac{2N}{t}\right)^{\frac{4\alpha}{3}} + \sqrt[3]{2}\left(\frac{t}{2N}\right)^{\frac{2\alpha}{3}}\right]^{\frac{1}{2}} = \frac{2\sqrt{3}}{\sqrt[3]{2}}\left(\frac{t}{2N}\right)^{\frac{\alpha}{3}}.$$

Hence

$$\begin{aligned}\int_T (\tau(\xi(t)-\xi_\Lambda(t)))^p dt &\leqslant \left(\frac{2\sqrt{3}}{\sqrt[3]{2}}\right)^p\left(\frac{1}{2N}\right)^{\frac{\alpha p}{3}}\int_T t^{\frac{\alpha p}{3}}dt \\ &= \left(\frac{2\sqrt{3}}{\sqrt[3]{2}}\right)^p\left(\frac{1}{2N}\right)^{\frac{\alpha p}{3}}\frac{A^{\frac{\alpha p}{3}+1}}{\frac{\alpha p}{3}+1} =: y_2(\alpha).\end{aligned}$$

In turn, minimizing $y_2(\alpha)$ in α, we deduce

$$y_2(\alpha_0), \qquad \alpha_0 = \frac{3(1-\ln\frac{A}{2N})}{p\ln\frac{A}{2N}}.$$

Thus

$$\begin{aligned}\int_T (\tau(\xi(t)-\xi_\Lambda(t)))^p dt &\leqslant \left(\frac{2\sqrt{3}}{\sqrt[3]{2}}\right)^p\left(\frac{A}{2N}\right)^{\frac{1}{\ln\frac{A}{2N}}} 2N\ln\frac{A}{2N} \\ &= 2\left(\frac{2\sqrt{3}}{\sqrt[3]{2}}\right)^p\left(e^{\ln\frac{A}{2N}}\right)^{\frac{1}{\ln\frac{A}{2N}}} N\ln\frac{A}{2N} = 2\left(\frac{2\sqrt{3}}{\sqrt[3]{2}}\right)^p eN\ln\frac{A}{2N}.\end{aligned}$$

Hence, by Theorem 3.2, the inequality

$$\int_T (\tau(\xi(t)-\xi_\Lambda(t)))^p\, dt \leqslant \frac{\varepsilon^p}{\max\left(p^{\frac{p}{2}}, \left(2\ln\frac{2}{\delta}\right)^{\frac{p}{2}}\right)},$$

follows, when N satisfies

$$N \ln \frac{A}{2N} \leqslant \frac{\varepsilon^p}{2\left(\frac{2\sqrt{3}}{\sqrt[3]{2}}\right)^p \cdot e \cdot \max\left(p^{\frac{p}{2}}, \left(2\ln\frac{2}{\delta}\right)^{\frac{p}{2}}\right)}. \qquad (3)$$

Thus the model $\xi_\Lambda(t)$ approximate process $\xi(t)$ with accuracy $\varepsilon > 0$ and reliability $1-\delta, 0 < \delta < 1$ in the space $L_p(T)$ under condition (3).

4. Conclusion

In this paper we constructed the model of Gaussian stochastic process by a new approach which generalizes the spectral simulation method. We also found conditions under which this model approximates the process with a given accuracy and reliability in the space $L_p(T)$, $p \geqslant 1$.

REFERENCES

[1] R. G. ANTONINI, YU. V. KOZACHENKO, A. M. TEGZA, *Accuracy of simulation in L_p Gaussian random processes*, Vīsn. Kiïv Unīv. Ser. Fīz.–Mat. Nauki **5** (2002), 7–14.
[2] V. V. BULDYGIN, YU. V. KOZACHENKO, *Metric Characterization of Random Variables and Random Processes*, Translated from the 1998 Russian original by V. Zaiats. Translations of Mathematical Monographs, 188. American Mathematical Society, Providence, RI, 2000.
[3] S. M. ERMAKOV, G. A. MYKHAYLOV, *Statistical Modeling*, Second edition, Nauka, Moscow, 1982. (in Russian)
[4] J. P. KAHANE, *Propriétés locales des fonctions à séries de Fourier aléatories*, Studia Math. **19 (1)** (1960), 1–25.
[5] K. KARHUNEN, *Zur Spektraltheorie stochastischer Prozesse*, Ann. Acad. Sci. Fennicae. Ser. A. I. Math.-Phys. **34** (1946), 7 pp.
[6] YU. V. KOZACHENKO, O. E. KAMENSHCHIKOVA, *Approximation of* $\mathrm{SSub}_\varphi(\Omega)$ *random processes in* $L_p(\mathbb{T})$, Theor. Probab. Math. Statist. **79** (2009), 83–88.
[7] YU. KOZACHENKO, A. OLENKO, O. POLOSMAK, *Convergence in $L_p([0,T])$ of wavelet expansions of φ-sub-Gaussian random processes*, Methodol. Comput. Appl. Probab. (2013), (to appear). [DOI:10.1007/s11009-013-9346-7]
[8] YU. KOZACHENKO, I. ROZORA, *Simulation of stochastic processes*, Random Oper. Stochastic Equations **11(3)** (2003), 275–296.
[9] K. K. SABEL'FEL'D, O. A. KURBANMURADOV, *Numerical statistical model of classical incompressible isotropic turbulence*, Soviet J. Numer. Anal. Math. Modelling **5 (3)** (1990), 251–263.

(Received November 14, 2013)

Nataliya Troshki
Taras Shevchenko National University of Kyiv
Department of Probability Theory
Statistics and Actuarial Mathematics
64, Volodymyrs'ka St., Kyiv 01033, Ukraine
e-mail: FedoryanichNatali@ukr.net