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GENERALIZED IDEAL CONVERGENCE IN PROBABILISTIC NORMED SPACES

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Abstract. The aim of this paper is to introduce and study the notion of I_λ-convergence in probabilistic normed space as a variant of the notion of ideal convergence. Also I_λ-limit point and I_λ-cluster point hase been defined and the relation between them have been established. Finally, we establish example which shows that our method of convergence on probabilistic normed space is more general.

1. Introduction

The notion of statistical convergence for sequences of real numbers was introduced by Steinhaus [30] and Fast [6] independently. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory and number theory. Later on it was further investigated from various points of view. For example, statistical convergence has been investigated in summability theory by (Connor [4], Fridy [7], Šalát [25]), number theory and mathematical analysis by (Buck [2]), topological spaces (Di Maio and Kočinac [21]), function spaces (Caserta, Di Maio and Kočinac [3]), locally convex spaces (Maddox [20]).

Menger [22] proposed the probabilistic concept of the distance by replacing the number $d(p,q)$ as the distance between points p,q by a probability distribution function $F_{p,q}(x)$. He interpreted $F_{p,q}(x)$ as the probability that the distance between p and q is less than x. This led to the development of the area now called probabilistic metric space. Šerstnev [29] who first used this idea of Menger to introduce the concept of a probabilistic normed space. In 1993, Alsina et al. [1] presented a new definition of probabilistic normed space which includes the definition of Šerstnev as a special case. For an extensive view on this subject, we refer to [8, 9, 11, 16, 27, 28]. Subsequently, Mursaleen and Mohiuddine [23] and Rahmat [24] studied the ideal convergence in probabilistic normed spaces and Kumar and Kumar [15] studied I-Cauchy and I^*-Cauchy sequences probabilistic normed space.

The notion of statistical convergence depends on the density (asymptotic or natural) of subsets of $\mathbb{N}$. A subset E of $\mathbb{N}$ is said to have natural density $\delta(E)$ if

$$\delta(E) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_E(k)$$

exists, where χ_E is the *characteristic function* of E.

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DEFINITION 1.1. A sequence $x = (x_k)$ is said to be *statistically convergent* to ℓ if for every $\varepsilon > 0$
$$\delta(\{k \in \mathbb{N} : |x_k - \ell| \geqslant \varepsilon\}) = 0.$$
In this case, we write $S - \lim x = \ell$ or $x_k \to \ell(S)$.

The notion of I-convergence was initially introduced by Kostyrko et al., [13] as a generalization of statistical convergence which is based on the structure of the ideal I of subsets of natural numbers $\mathbb{N}$. Kostyrko et al., [14] gave some of basic properties of I-convergence and dealt with extremal I-limit points. For an extensive view on this subject, we refer to [5, 10, 17, 26, 31], and reference therein.

Although an ideal is defined as a hereditary and additive family of subsets of a non-empty arbitrary set X, here in our study it suffices to take I as a family of subsets of $\mathbb{N}$, positive integers, i.e. $I \subset 2^{\mathbb{N}}$, such that $\phi \in I$, $A \cup B \in I$ for each $A, B \in I$, and each subset of an element of I is an element of I. A non-empty family of sets $F \subset 2^{\mathbb{N}}$ is a filter on $\mathbb{N}$ if and only if $\phi \notin F$, $A \cap B \in F$ for each $A, B \in F$, and any superset of an element of F is in F. An ideal I is called *non-trivial* if $I \neq \phi$ and $\mathbb{N} \notin I$. Clearly I is a non-trivial ideal if and only if $F = F(I) = \{\mathbb{N} - A : A \in I\}$ is a filter in $\mathbb{N}$, called the filter associated with the ideal I. A non-trivial ideal I is called *admissible* if and only if $\{\{n\} : n \in \mathbb{N}\} \subset I$. A non-trivial ideal I is maximal if there cannot exists any non-trivial ideal $J \neq I$ containing I as a subset. Recall from [13] that a sequence $x = (x_k)$ of points in $\mathbb{R}$ is said to be I-convergent to a real number ℓ if $\{k \in \mathbb{N} : |x_k - \ell| \geqslant \varepsilon\} \in I$ for every $\varepsilon > 0$. In this case we write $I - \lim x_k = \ell$.

2. Basic definitions and notations

Now we recall some notations and basic definitions that we are going to use in this paper.

DEFINITION 2.1. A *distribution function* (briefly a d.f.) F is a function from the extended reals $(-\infty, +\infty)$ into $[0, 1]$ such that

(a) it is non-decreasing ;

(b) it is left-continuous on $(-\infty, +\infty)$;

(c) $F(-\infty) = 0$ and $F(+\infty) = 1$.

The set of all d.f.'s will be denoted by Δ. The subset of Δ consisting of proper d.f's, namely of those elements F such that $\ell^+ F(-\infty) = F(-\infty) = 0$ and $\ell^- F(+\infty) = F(+\infty) = 1$ will be denoted by D. A *distance distribution function* (briefly, d.d.f.) is a d.f. F such that $F(0) = 0$. The set of all d.d.f's will be denoted by Δ^+, while $D^+ := D \cap \Delta^+$ will denote the set of proper d.d.f.'s.

DEFINITION 2.2. [12] A *triangular norm* or, briefly, a *t-norm* is a binary operation $T : [0, 1] \times [0, 1] \to [0, 1]$ that satisfies the following conditions:

(T1) T is commutative, i.e., $T(s, t) = T(t, s)$ for all s and t in $[0, 1]$;

(T2) T is associative, i.e., $T(T(s, t), u) = T(s, T(t, u))$ for all s, t and u in $[0, 1]$;

(T3) T is nondecreasing, i.e., $T(s,t) \leqslant T(s',t)$ for all $t \in [0,1]$ whenever $s \leqslant s'$;

(T4) T satisfies the boundary condition $T(1,t) = t$ for every $t \in [0,1]$.

T^* is a continuous *t-conorm*, namely, a continuous binary operation on $[0,1]$ that is related to a continuous t-norm through $T^*(s,t) = 1 - T(1-s, 1-t)$. Notice that by virtue of its commutativity, any t-norm T is nondecreasing in each place. Some examples of t-norms T and its t-conorms T^* are: $M(x,y) = \min\{x,y\}$, $\Pi(x,y) = x.y$ and $M^*(x,y) = \max\{x,y\}$, $\Pi^*(x,y) = x + y - x.y$.

Using the definitions just given above Šerstnev [29] defined a probabilistic normed space as follows:

DEFINITION 2.3.. A triplet (X, ν, T) is called a *probabilistic normed space* (in short PNS) if X is a real vector space, ν is a mapping from X into D and for $x \in X$, the d.f. $\nu(x)$ is denoted by ν_x, $\nu_x(t)$ is the value of ν_x at $t \in \mathbb{R}$ and T is a *t-norm.* ν satisfies the following conditions:

(i) $\nu_x(0) = 0$;

(ii) $\nu_x(t) = 1$ for all $t > 0$ if and only if $x = 0$;

(iii) $\nu_{ax}(t) = \nu_x\left(\frac{t}{|a|}\right)$ for all $a \in \mathbb{R} \diagdown \{0\}$;

(iv) $\nu_{x+y}(s+t) \geqslant T(\nu_x(s), \nu_y(t))$ for all $x, y \in X$ and $s, t \in \mathbb{R}_0^+$.

Let $(X, ||.||)$ be a normed space and $\mu \in D$ with $\mu(0) = 0$ and $\mu \neq \varepsilon_0$, where

$$\varepsilon_0(t) = \begin{cases} 0, & \text{if } t \leqslant 0 \\ 1, & \text{if } t > 0 \end{cases}$$

For $x \in X$, $t \in \mathbb{R}$, if we define

$$\nu_x(t) = \mu\left(\frac{t}{||x||}\right), \quad x \neq 0,$$

then in [18], it is proved that (X, ν, T) is a probabilistic normed space in the sense of Definition 2.3.

DEFINITION 2.4. Let (X, ν, T) be a PNS and $x = (x_k)$ be a sequence in X. We say that (x_k) is *convergent* to $\ell \in X$ with respect to the probabilistic norm ν if for each $\varepsilon > 0$ and $\alpha \in (0,1)$ there exists a positive integer m such that $\nu_{x_k - \ell}(\varepsilon) > 1 - \alpha$ whenever $k \geqslant m$. The element ℓ is called the ordinary limit of the sequence (x_k) and we shall write $\nu - \lim x_k = \ell$ or $x_k \xrightarrow{\nu} \ell$ as $k \to \infty$.

DEFINITION 2.5. Let (X, ν, T) be a PNS. A sequence (x_k) in X is said to be *Cauchy* with respect to the probabilistic norm ν if for each $\varepsilon > 0$ and $\alpha \in (0,1)$ there exist a positive integer $M = M(\varepsilon, \alpha)$ such that $\nu_{x_k - x_p}(\varepsilon) > 1 - \alpha$ whenever $k, p \geqslant M$.

DEFINITION 2.6. Let (X, ν, T) be a PNS, and let $r \in (0,1)$ and $x \in X$. The set

$$B(x, r; t) = \left\{ y \in X : \nu_{y-x}(t) > 1 - r \right\}$$

is called open ball with center x and radius r with respect to t.

DEFINITION 2.7. [19] Let $\lambda = (\lambda_n)$ be a non-decreasing sequence of positive numbers tending to infinity such that $\lambda_{n+1} \leqslant \lambda_n + 1$, $\lambda_1 = 1$. The generalized de la Vallée-Poussin mean of a sequence (x_k) is defined by

$$\sigma_n(x) = \frac{1}{\lambda_n} \sum_{k \in I_n} x_k$$

where $J_n = [n - \lambda_n + 1, n]$.

A sequence $x = (x_k)$ is said to be (V, λ)-summable to number L, if $\sigma_n(x) \to L$ as $n \to \infty$. In this case L is called λ*-limit* of x. If $\lambda_n = n$, then (V, λ)-summability reduces to $(C, 1)$-summability.

Throughout the paper, we assume I is an admissible ideal of subsets of $\mathbb{N}$.

3. Main results

In this section we introduced the notions of λ-convergence and I_λ-convergence and proved some new results related to these notions.

DEFINITION 3.1. Let (X, ν, T) be a PNS. A sequence $x = (x_k)$ in X is λ-convergent to $L \in X$ with respect to the probabilistic norm ν if, for $\alpha \in (0,1)$ and every $\varepsilon > 0$, there exists $n_o \in \mathbb{N}$ such that

$$\nu_{\sigma_n(x)-L}(\varepsilon) > 1 - \alpha$$

for all $n \geqslant n_o$, where

$$\sigma_n(x) = \frac{1}{\lambda_n} \sum_{k \in J_n} x_k.$$

In this case, we write $\nu^\lambda - \lim x = L$.

DEFINITION 3.2. Let $I \subset 2^{\mathbb{N}}$ and (X, ν, T) be a PNS. A sequence $x = (x_k)$ in X is said to be I_λ-convergent to $L \in X$ with respect to the probabilistic norm ν if, for every $\varepsilon > 0$ and $\alpha \in (0,1)$ the set

$$\left\{n \in \mathbb{N} : \ \nu_{\sigma_n(x)-L}(\varepsilon) \leqslant 1 - \alpha\right\} \in I.$$

L is called the I_λ-limit of the sequence $x = (x_k)$ in X, and we write $I_\lambda^\nu - \lim x = L$.

EXAMPLE 3.1. Let $(\mathbb{R}, |.|)$ denote the space of all real numbers with the usual norm, and let $T(a,b) = ab$ for all $a, b \in [0,1]$. For all $x \in \mathbb{R}$ and every $t > 0$, consider $\nu_x(t) = \frac{t}{t+|x|}$. Then $(\mathbb{R}, \nu, T)$ is a PNS. If we take $I = \{A \subset \mathbb{N} : \ \delta(A) = 0\}$, where $\delta(A)$ denotes the natural density of the set A, then I is a non-trivial admissible ideal. Define a sequence $x = (x_k)$ as follows:

$$x_k = \begin{cases} 1, & \text{if } k = i^2, i \in \mathbb{N} \\ 0, & \text{otherwise.} \end{cases}$$

Then for every $\alpha \in (0,1)$ and for any $\varepsilon > 0$, the set

$$K = \left\{ n \in \mathbb{N} : \ \nu_{\sigma_n(x)}(\varepsilon) \leqslant 1 - \alpha \right\}$$

is finite. Hence $\delta(K) = 0$ and consequently $K \in I$, i.e., $I_\lambda^\nu - \lim x = 0$.

LEMMA 3.1. *Let (X, ν, T) be a PNS and $x = (x_k)$ be a sequence in X. Then, for every $\varepsilon > 0$ and $\alpha \in (0,1)$ the following statements are equivalent:*

(i) $I_\lambda^\nu - \lim x = L$,

(ii) $\left\{ n \in \mathbb{N} : \ \nu_{\sigma_n(x)-L}(\varepsilon) \leqslant 1 - \alpha \right\} \in I$

(iii) $I_\lambda - \lim \nu_{x_k - L}(\varepsilon) = 1$.

THEOREM 3.1. *Let (X, ν, T) be a PNS and if a sequence $x = (x_k)$ in X is I_λ-convergent to $L \in X$ with respect to the probabilistic norm ν, then $I_\lambda^\nu - \lim x$ is unique.*

Proof. Suppose that $I_\lambda^\nu - \lim x = L_1$ and $I_\lambda^\nu - \lim x = L_2$ $(L_1 \neq L_2)$. Given $\alpha > 0$ and choose $\beta \in (0,1)$ such that

$$T(1-\beta, 1-\beta) > 1 - \alpha. \tag{3.1}$$

Then for $\varepsilon > 0$, define the following sets:

$$K_1 = \left\{ n \in \mathbb{N} : \ \nu_{\sigma_n(x)-L_1}\left(\frac{\varepsilon}{2}\right) \leqslant 1 - \beta \right\},$$

$$K_2 = \left\{ n \in \mathbb{N} : \ \nu_{\sigma_n(x)-L_2}\left(\frac{\varepsilon}{2}\right) \leqslant 1 - \beta \right\},$$

Since $I_\lambda^\nu - \lim x = L_1$, using Lemma 2.1, we have $K_1 \in I$. Also, using $I_\lambda^\nu - \lim x = L_2$, we get $K_2 \in I$. Now let

$$K = K_1 \cup K_2.$$

Then $K \in I$. This implies that its complement K^c is a non-empty set in $F(I)$. Now if $n \in K^c = K_1^c \cap K_2^c$, we have

$$\nu_{\sigma_n(x)-L_1}\left(\frac{\varepsilon}{2}\right) > 1 - \beta \text{ and } \nu_{\sigma_n(x)-L_2}\left(\frac{\varepsilon}{2}\right) > 1 - \beta.$$

Now, we choose a $s \in \mathbb{N}$ such that

$$\nu_{x_s - L_1}\left(\frac{\varepsilon}{2}\right) > \nu_{\sigma_n(x)-L_1}\left(\frac{\varepsilon}{2}\right) > 1 - \beta$$

and

$$\nu_{x_s - L_2}\left(\frac{\varepsilon}{2}\right) > \nu_{\sigma_n(x)-L_2}\left(\frac{\varepsilon}{2}\right) > 1 - \beta$$

Then from (2.1), we have

$$\nu_{L_1 - L_2}(\varepsilon) \geqslant T\left(\nu_{x_s - L_1}\left(\frac{\varepsilon}{2}\right), \qquad \nu_{x_s - L_2}\left(\frac{\varepsilon}{2}\right)\right) > T(1-\beta, 1-\beta) > 1 - \alpha.$$

Since $\alpha > 0$ is arbitrary, we have $\nu_{L_1 - L_2}(\varepsilon) = 1$ for all $\varepsilon > 0$, which implies that $L_1 = L_2$. Therefore, we conclude that $I_\lambda^\nu - \lim x$ is unique. $\square$

THEOREM 3.2. *Let (X,ν,T) be a PNS and let $x=(x_k)$ be a sequence in X. If $\nu^{\lambda}-\lim x=L$, then $I_{\lambda}^{\nu}-\lim x=L$.*

Proof. Let $\nu^{\lambda}-\lim x=L$, then for every $\varepsilon>0$ and given $\alpha\in(0,1)$, there exists $n_0\in\mathbb{N}$ such that

$$\nu_{\sigma_n(x)-L}(\varepsilon)>1-\alpha$$

for all $n\geqslant n_0$. Therefore the set

$$B=\left\{n\in\mathbb{N}:\ \nu_{\sigma_n(x)-L}(\varepsilon)\leqslant 1-\alpha\right\}\subseteq\{1,2,...,n_0-1\}.$$

But, with I being admissible, we have $B\in I$. Hence $I_{\lambda}^{\nu}-\lim x=L$. □

COROLLARY 3.3. *Let (X,ν,T) be a PNS and let $x=(x_k)$ be a sequence in X. If $x=(x_k)$ is λ-convergent with respect to the probabilistic norm ν, then $\nu^{\lambda}-\lim x$ is unique.*

Proof. Suppose that $\nu^{\lambda}-\lim x=L_1$ and $\nu^{\lambda}-\lim x=L_2$ $(L_1\neq L_2)$. Given $\alpha\in(0,1)$ and choose $\beta\in(0,1)$ such that $T(1-\beta,1-\beta)>1-\alpha$. Then for any $\varepsilon>0$, there exists $n_1\in\mathbb{N}$ such that

$$\nu_{\sigma_n(x)-L_1}(\varepsilon)>1-\alpha$$

for all $n\geqslant n_1$. Also, there exists $n_2\in\mathbb{N}$ such that

$$\nu_{\sigma_n(x)-L_2}(\varepsilon)>1-\alpha$$

for all $n\geqslant n_2$. Now, consider $n_o=\max\{n_1,n_2\}$. Then for $n\geqslant n_o$, we will get a $s\in\mathbb{N}$ such that

$$\nu_{x_s-L_1}\left(\frac{\varepsilon}{2}\right)>\nu_{\sigma_n(x)-L_1}\left(\frac{\varepsilon}{2}\right)>1-\beta$$

and

$$\nu_{x_s-L_2}\left(\frac{\varepsilon}{2}\right)>\nu_{\sigma_n(x)-L_2}\left(\frac{\varepsilon}{2}\right)>1-\beta.$$

Then, we have

$$\nu_{L_1-L_2}(\varepsilon)\geqslant T\left(\nu_{x_s-L_1}\left(\frac{\varepsilon}{2}\right),\qquad \nu_{x_s-L_2}\left(\frac{\varepsilon}{2}\right)\right)>T(1-\beta,1-\beta)>1-\alpha.$$

Since $\alpha>0$ is arbitrary, we have $\nu_{L_1-L_2}(\varepsilon)=1$ for all $\varepsilon>0$, which implies that $L_1=L_2$. □

A method G is called regular if every convergent sequence $\mathbf{x}=(x_n)$ is G-convergent with $G(\mathbf{x})=\lim\mathbf{x}$. A method is called subsequential if whenever $\mathbf{x}$ is G-convergent with $G(\mathbf{x})=\ell$, then there is a subsequence (x_{n_k}) of $\mathbf{x}$ with $\lim_k x_{n_k}=\ell$. Since the ordinary convergence implies ideal convergence, so I is a regular sequential method.

THEOREM 3.4. *Sequential method I_{λ} is regular, i.e. If $\nu^{\lambda}-\lim x=L$, then $I_{\lambda}^{\nu}-\lim x=L$.*

Proof. The proof follows from the fact that I is admissible and Theorem 3.2. □

THEOREM 3.5. *Let (X,ν,T) be a PNS and let $x=(x_k)$ be sequence in X. If $\nu^{\lambda}-\lim x=L$, then there exists a subsequence $\left(x_{m_k}\right)$ of $x=(x_k)$ such that $\nu-\lim x_{m_k}=L$.*

Proof. Let $\nu^{\lambda}-\lim x=L$. Then, for every $\varepsilon>0$ and given $\alpha\in(0,1)$, there exists $n_0\in\mathbb{N}$ such that

$$\nu_{\sigma_n(x)-L}(\varepsilon)>1-\alpha$$

for all $n\geqslant n_0$. Clearly, for each $n\geqslant n_0$, we can select an $m_k\in J_n$ such that

$$\nu_{x_{m_k}-L}(\varepsilon)>\nu_{\sigma_n(x)-L}(\varepsilon)>1-\alpha.$$

It follows that $\nu-\lim x_{m_k}=L$. $\square$

DEFINITION 3.3. Let (X,ν,T) be a PNS and let $x=(x_k)$ be a sequence in X. Then,

(1) An element $L\in X$ is said to be I_{λ}-limit point of $x=(x_k)$ if there is a set $M=\{m_1<m_2<...<m_k<...\}\subset\mathbb{N}$ such that the set $M^{\iota}=\{n\in\mathbb{N}:\ m_k\in J_n\}\notin I$ and $\nu^{\lambda}-\lim x_{m_k}=L$.

(2) An element $L\in X$ is said to be I_{λ}-cluster point of $x=(x_k)$ if for every $\varepsilon>0$ and $\alpha\in(0,1)$, the set

$$\left\{n\in\mathbb{N}:\ \nu_{\sigma_n(x)-L}(\varepsilon)>1-\alpha\right\}\notin I.$$

Let $\Lambda_{\nu}^{I_{\lambda}}(x)$ denote the set of all I_{λ}-limit points and $\Gamma_{\nu}^{I_{\lambda}}(x)$ denote the set of all I_{λ}-cluster points in X, respectively.

THEOREM 3.6. *Let (X,ν,T) be a PNS. For each sequence $x=(x_k)$ in X, then $\Lambda_{\nu}^{I_{\lambda}}(x)\subset\Gamma_{\nu}^{I_{\lambda}}(x)$.*

Proof. Let $L\in\Lambda_{\nu}^{I_{\lambda}}(x)$. Then there exists a set $M\subset\mathbb{N}$ such that $M^{\iota}\notin I$, where M and M^{ι} are as in Definition 3.3., satisfies $\nu^{\lambda}-\lim x_{m_k}=L$. Thus, for every $\varepsilon>0$ and $\alpha\in(0,1)$, there exists $n_0\in\mathbb{N}$ such that

$$\nu_{\sigma_n'(x)-L}(\varepsilon)>1-\alpha$$

for all $n\geqslant n_0$, where $\sigma_n'(x)=\frac{1}{\lambda_n}\sum_{k\in J_n}x_{m_k}$. Therefore,

$$B=\left\{n\in\mathbb{N}:\ \nu_{\sigma_n'(x)-L}(\varepsilon)>1-\alpha\right\}\supseteq M^{\iota}\setminus\left\{m_1,m_2,...,m_{n_0}\right\}.$$

Now, with I being admissible, we must have $M^{\iota}\setminus\left\{m_1,m_2,...,m_{k_0}\right\}\notin I$ and as such $B\notin I$. Hence $L\in\Gamma_{\nu}^{I_{\lambda}}(x)$. $\square$

THEOREM 3.7. *Let (X,ν,T) be a PNS. For each sequence $x=(x_k)$ in X, the set $\Gamma_{\nu}^{I_\lambda}(x)$ is closed set in X with respect to the usual topology induced by the probabilistic norm ν^{λ}.*

Proof. Let $y\in\overline{\Gamma_{\nu}^{I_\lambda}(x)}$. Take $\varepsilon>0$ and $\alpha\in(0,1)$. Then there exists $L_0\in\Gamma_{\nu}^{I_\lambda}(x)\cap B(y,\alpha,\varepsilon)$. Choose $\delta>0$ such that $B(L_0,\delta,\varepsilon)\subset B(y,\alpha,\varepsilon)$. We have

$$G=\left\{n\in\mathbb{N}:\ \nu_{\sigma_n(x)-y}(\varepsilon)>1-\alpha\right\}$$

$$\supseteq\left\{n\in\mathbb{N}:\ \nu_{\sigma_n(x)-L_0}(\varepsilon)>1-\delta\right\}=H.$$

Thus $H\notin I$ and so $G\notin I$. Hence $y\in\Gamma_{\nu}^{I_\lambda}(x)$. □

THEOREM 3.8. *Let (X,ν,T) be a PNS and let $x=(x_k)$ in X. Then the following statements are equivalent:*

(1) L is a I_λ-limit point of x,

(2) There exist two sequences y and z in X such that $x=y+z$ and $\nu^{\lambda}-\lim y=L$ and $\{n\in\mathbb{N}:\ k\in J_n,\ z_k\neq\theta\}\in I$, where θ is the zero element of X.

Proof. Suppose that (1) holds. Then there exist sets M and M^{ι} as in Definition 3.3. such that $M^{\iota}\notin I$ and $\nu^{\lambda}-\lim x_{m_k}=L$. Define the sequences y and z as follows:

$$y_k=\begin{cases} x_k, & \text{if } k\in J_n;\, n\in M^{\iota},\\ L, & \text{otherwise.}\end{cases}$$

and

$$z_k=\begin{cases} \theta, & \text{if } k\in J_n;\, n\in M^{\iota},\\ x_k-L, & \text{otherwise.}\end{cases}$$

It sufficies to consider the case $k\in J_n$ such that $n\in\mathbb{N}\setminus M^{\iota}$. Then for each $\alpha\in(0,1)$ and $\varepsilon>0$, we have $\nu_{y_k-L}(\varepsilon)=1>1-\alpha$. Thus, in this case,

$$\nu_{\sigma_n(y)-L}(\varepsilon)=1>1-\alpha.$$

Hence $\nu^{\lambda}-\lim y=L$.

Now $\{n\in\mathbb{N}:\ k\in J_n,\ z_k\neq\theta\}\subset\mathbb{N}\setminus M^{\iota}$ and so $\{n\in\mathbb{N}:\ k\in J_n,\ z_k\neq\theta\}\in I$.

Now, suppose that (2) holds. Let $M^{\iota}=\{n\in\mathbb{N}:\ k\in J_n,\ z_k=\theta\}$. Then, clearly $M^{\iota}\in F(I)$ and so it is an infinite set. Construct the set $M=\{m_1<m_2<...<m_k<...\}\subset\mathbb{N}$ such that $m_k\in J_n$ and $z_{m_k}=\theta$. Since $x_{m_k}=y_{m_k}$ and $\nu^{\lambda}-\lim y=L$ we obtain $\nu^{\lambda}-\lim x_{m_k}=L$. This completes the proof. □

THEOREM 3.9. *Let (X,ν,T) be a PNS and $x=(x_k)$ be a sequence in X. Let I be a non-trivial ideal in $\mathbb{N}$. If there is a I_{λ}^{ν}-convergent sequence $y=(y_k)$ in X such that $\{k\in\mathbb{N}:y_k\neq x_k\}\in I$ then x is also I_{λ}^{ν}-convergent.*

Proof. Suppose that $\{k\in\mathbb{N}:y_k\neq x_k\}\in I$ and $I_{\lambda}^{\nu}-\lim y=L$. Then for every $\alpha\in(0,1)$ and $\varepsilon>0$, the set

$$\left\{n\in\mathbb{N}:\ \nu_{\sigma_n(y)-L}(\varepsilon)\leqslant 1-\alpha\right\}\in I.$$

For every $0<\alpha<1$ and $\varepsilon>0$, we have

$$\left\{n\in\mathbb{N}:\ \nu_{\sigma_n(x)-L}(\varepsilon)\leqslant 1-\alpha\right\} \tag{3.2}$$

$$\subseteq\{k\in\mathbb{N}:y_k\neq x_k\}\cup\left\{n\in\mathbb{N}:\ \nu_{\sigma_n(y)-L}(\varepsilon)\leqslant 1-\alpha\right\}.$$

As both the sets of right-hand side of (3.2) are in I, we have that

$$\left\{n\in\mathbb{N}:\ \nu_{\sigma_n(x)-L}(\varepsilon)\leqslant 1-\alpha\right\}\in I.$$

This completes the proof of the theorem. □

REFERENCES

[1] C. ALSINA, B. SCHWEIZER, A. SKLAR, *On the definition of a probabilistic normed space*, Aequationes Math., **46**, 91–98 (1993).
[2] R. C. BUCK, *The measure theoretic approach to density*, Amer. J. Math. **68** (1946) 560–580.
[3] A. CASERTA, G. DI MAIO, LJ. D. R. KOČINAC, *Statistical convergence in function spaces*, Abstr. Appl. Anal. Vol. 2011 (2011), Article ID 420419, 11 pages.
[4] J. CONNOR, *The statistical and strong p-Cesáro convergence of sequences*, Analysis **8** (1988) 47–63.
[5] K. DEMS, *On I-Cauchy sequences*, Real Anal. Exchange, **30** (1) (2004/2005), 123–128.
[6] H. FAST, *Sur la convergence statistique*, Colloq. Math. **2** (1951) 241–244.
[7] J. A. FRIDY, *On statistical convergence*, Analysis, **5** (1985) 301–313.
[8] B. L. GUILLÉN, J. A. R. LALLENA, C. SEMPI, *A study of boundedness in probabilistic normed spaces*, J. Math. Anal. Appl. **232** (1999), 183–196.
[9] B. L. GUILLÉN, C. SEMPI, *Probabilistic norms and convergence of random variables*, J. Math. Anal. Appl. **280** (2003), 9–16.
[10] B. HAZARIKA, *On generalized difference ideal convergence in random 2-normed spaces*, Filomat, **26** (6) (2012), 1265–1274.
[11] S. KARAKUŞ, *Statistical convergence on probabilistic normed spaces*, Math. Comm. **12** (2007), 11–23.
[12] E. P. KLEMENT, R. MESIAR, E. PAP, *Triangular Norms*, Kluwer, Dordrecht, 2000.
[13] P. KOSTYRKO, T. ŠALÁT, W. WILCZYŃSKI, I-convergence, Real Anal. Exchange **26** (2) (2000–2001), 669–686.
[14] P. KOSTYRKO, M. MAČAJ, T. ŠALÁT, M. SLEZIAK, *I-convergence and Extremal I-limit points*, Math. Slovaca 2005, **55**, 443–64.
[15] K. KUMAR, V. KUMAR, *On the I and I^*-Cauchy sequences in probabilistic normed spaces*, Mathematical Sciences, **2** (1), 47–58 (2008).
[16] V. KUMAR, B. L. GUILLÉN, *On Ideal Convergence of Double Sequences in Probabilistic Normed Spaces*, Acta Math. Sinica, English Series, **28** (8) (2012), 1689–1700.
[17] B. K. LAHIRI, P. DAS, *I and I^*-convergence in topological spaces*, Math. Bohemica, **130** (2005), 153–160.
[18] B. LAFUERZA-GUILLÉN, J. A. RODRÍGUEZ-LALLENA, C. SEMPI, *Some classes of probabilistic normed spaces*, Rend. Mat., **17** (1997), 237–252.
[19] L. LEINDLER, *Über die de la Vallee-Pousinsche Summeierbarkeit allgemeiner Orthogonalreihen*, Acta Math. Acad. Sci. Hungar., **16** (1965), 375–387.
[20] I. J. MADDOX, *Statistical convergence in a locally convex spaces*, Math. Proc. Cambridge Philos. Soc., **104** (1) (1988), 141–145.
[21] G. DI MAIO, LJ. D. R. KOČINAC, *Statistical convergence in topology*, Topology Appl. **156**, (2008), 28–45.
[22] K. MENGER, *Statistical metrics*, Proc. Nat. Acad. Sci. USA, **28** (1942), 535–537.
[23] M. MURSALEEN, S. A. MOHIUDDINE, *On ideal convergence in probabilistic normed spaces*, Math. Slovaca, **62** (1) (2012), 49–62.

[24] M. R. S. RAHMAT, *Ideal Convergence on Probabilistic Normed Spaces*, Inter. Jour. Stat. Econ., **3** (9) (2009), 67–75.
[25] T. ŠALÁT, *On statistical convergence of real numbers*, Math. Slovaca, **30** (1980), 139–150.
[26] T. ŠALÁT, B. C. TRIPATHY, M. ZIMAN, *On some properties of I-convergence*, Tatra Mt. Math. Publ. 2004, **28**, 279–86.
[27] B. SCHWEIZER, A. SKLAR, *Statistical metric spaces*, Pacific J. Math. **10** (1960), 313–334.
[28] B. SCHWEIZER, A. SKLAR, *Probabilistic Metric Spaces*, North Holland, New York-Amsterdam-Oxford, 1983.
[29] A. N. ŠERSTNEV, *Random normed spaces: Problems of completeness*, Kazan Gos. Univ. Ucen. Zap., **122**, 3–20 (1962).
[30] H. STEINHAUS, *Sur la convergence ordinaire et la convergence asymptotique*, Colloq. Math. **2** (1951), 73–74.
[31] B. C. TRIPATHY, B. HAZARIKA, *I-monotonic and I-convergent sequences*, Kyungpook Math. J. **51** (2011), 233–239, DOI 10.5666/KMJ.2011.51.2.233.

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