Journal of
Classical
Analysis
Volume 12, Number 2 (2018), 99–107
doi:10.7153/jca-2018-12-09

SHARP FEKETE–SZEGŐ COEFFICIENTS FUNCTIONAL, DISTORTION AND GROWTH INEQUALITIES FOR CERTAIN p–VALENT CLOSE–TO–CONVEX FUNCTIONS

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Abstract. In the present paper certain subclass $\mathscr{K}_p^s(\phi)$ of p-valent close-to-convex functions in the unit disc is defined by means of subordination. Sharp estimates for the Fekete-Szegő functional for functions belonging to the class $\mathscr{K}_p^s(\phi)$ are obtained. Sharp distortion theorem, growth theorem and a subordination result are also obtained.

1. Introduction and definitions

Let $\mathscr{A}_p$ denote the class of the functions of the form

$$f(z) = z^p + \sum_{n=1}^{\infty} a_{n+p} z^{n+p} \quad (p \in N), \tag{1}$$

which are p-valent analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. In particular, we write $\mathscr{A}_1 = \mathscr{A}$. If $f \in \mathscr{A}$ satisfies $f(z_1) \neq f(z_2)$ for any $z_1 \in \mathbb{U}$ and $z_2 \in \mathbb{U}$ with $z_1 \neq z_2$, then f is said to be univalent in $\mathbb{U}$ and denoted by $f \in \mathscr{S}$.

For any two analytic functions f and g in $\mathbb{U}$, we say that f is subordinate to g in $\mathbb{U}$, written as $f \prec g$ if there exists a Schwarz function w such that $f(z) = g(w(z))$ for $z \in \mathbb{U}$. In particular, if g is univalent in $\mathbb{U}$, then f is subordinate to g iff $f(0) = g(0)$ and $f(U) \subseteq g(U)$.

Let ϕ be an analytic function with positive real part in $\mathbb{U}$ with $\phi(0) = 1, \phi'(0) > 0$ and ϕ maps $\mathbb{U}$ onto a region starlike with respect to 1 which is symmetric with respect to the real axis. Let $\mathscr{S}_p^*(\phi)$ be the class of functions $f \in \mathscr{A}_p$ satisfying

$$\frac{1}{p}\frac{zf'(z)}{f(z)} \prec \phi(z) \quad (z \in \mathbb{U}) \tag{2}$$

and $\mathscr{C}_p(\phi)$ be the class of functions $f \in \mathscr{A}_p$ satisfying

$$\frac{1}{p}\left(1 + \frac{zf''(z)}{f'(z)}\right) \prec \phi(z) \quad (z \in \mathbb{U}). \tag{3}$$

Mathematics subject classification (2010): 30C45.
Keywords and phrases: Multivalent function, starlike function, close-to-convex function, Fekete-Szegő functional, distortion and growth theorem, subordination.

Paper JCA-12-09

These classes are studied by Ali et. al [2] and they obtained sharp distortion, growth, covering and rotation theorems for these classes. The classes $\mathscr{S}_p^*(\phi)$ and $\mathscr{C}_p(\phi)$ include several well known subclasses of p-valent starlike and p-valent convex function as special cases. In particular, for an analytic function $\phi(z)=\frac{1+(1-\frac{2\gamma}{p})z}{1-z}$, from (2), we obtain

$$\frac{zf'(z)}{f(z)} \prec \frac{p+(p-2\gamma)z}{1-z} \qquad (0 \leqslant \gamma < p,\ z \in \mathbb{U}). \tag{4}$$

Any function satisfying (4) belongs to class of p-valent starlike function of order γ denoted by $\mathscr{S}_p^*(\gamma)$. For $p=1$ the classes $\mathscr{S}_p^*(\phi)$ and $\mathscr{C}_p(\phi)$ are introduced and studied by Ma and Minda (see [17]). We denote that

$$\mathscr{S}_p^*(0)=\mathscr{S}_p^*, \quad \mathscr{S}_1^*(\gamma)=\mathscr{S}^*(\gamma) \quad \text{and} \quad \mathscr{S}_1^*(0)=\mathscr{S}^*.$$

A function $f \in \mathscr{A}_p$, is said to be p-valently close-to-convex of order γ $(0 \leqslant \gamma < p)$ in $\mathbb{U}$ if there exists a function $g \in \mathscr{S}_p^*(\gamma)$ and satisfies the inequality

$$\frac{zf'(z)}{g(z)} \prec \frac{p+(p-2\gamma)z}{1-z} \qquad (0 \leqslant \gamma < p,\ z \in \mathbb{U}). \tag{5}$$

The class of all p-valent close-to-convex functions of order γ in $\mathbb{U}$ is denoted by $\mathscr{K}_p(\gamma)$. Also, we denote that

$$\mathscr{K}_p(0)=\mathscr{K}_p, \quad \mathscr{K}_1(\gamma)=\mathscr{K}(\gamma) \quad \text{and} \quad \mathscr{K}_1(0)=\mathscr{K}.$$

In a recent paper Gao and Zhou [8] introduced an interesting subclass $\mathscr{K}_s$ of analytic and univalent function $f \in \mathscr{A}$ satisfying the following inequality:

$$Re\left(\frac{z^2 f'(z)}{g(z)g(-z)}\right) < 0 \quad (z \in \mathbb{U}).$$

for some $g \in \mathscr{S}^*\ (1/2)$. After that, many classes related to $\mathscr{K}_s$ investigated and studied by several authors. Especially, Wang et al. [24], Kowalczyk and Les-Bomba [12], Cho et al. [6], Xu et al. [25], Seker and Cho [22], Soni and Kant [21], Prajapat and Mishra [19] introduced the generalization of the class $\mathscr{K}_s$ and they obtained several properties of analytic functions in each classes.

Motivated essentially by the class $\mathscr{K}_s$ and the above referred works for analytic and univalent functions, we now introduce a new class of p-valent analytic functions in the following manner:

DEFINITION 1. Let ϕ be an analytic univalent function with positive real part in $\mathbb{U}$ with $\phi(0)=1$. The class $\mathscr{K}_p^s(\phi)$ consists of functions $f \in \mathscr{A}_p$ satisfying

$$\frac{1}{p}\left(\frac{(-1)^p z^{p+1} f'(z)}{g(z)g(-z)}\right) \prec \phi(z) \qquad (z \in \mathbb{U}) \tag{6}$$

for some function $g \in \mathscr{S}_p^*(p/2)$.

The bounds for Taylor coefficients of the function $f \in \mathscr{A}$ give information about the geometric properties of f. For example, if f is univalent in $\mathbb{U}$, then $|a_n| \leqslant n$ and the bounds for $|a_2|$ give the growth and distortion bounds for univalent functions. Some typical problems in geometric function theory are to study functionals made up of combinations of the coefficients of f. In 1933, Fekete and Szegő [7] obtained a sharp bound of the functional $\lambda a_2^2 - a_3$, with real λ $(0 \leqslant \lambda \leqslant 1)$ for a univalent function f. Since then, the problem of finding the sharp bounds for this functional of any compact family of functions $f \in \mathscr{A}$ with any complex λ is known as the classical Fekete-Szegő problem or inequality. In 1960 Lawrence Zalcman posed a conjecture that the coefficients of $\mathscr{S}$ satisfy the sharp inequality

$$|a_n^2 - a_{2n-1}| \leqslant (n-1)^2, \ \ n \geqslant 2.$$

More general versions of Zalcman conjecture have also been considered ([5, 14, 15, 16]) for the functional such as $\lambda a_n^2 - a_{2n-1}$ and $\lambda a_m a_n - a_{m+n-1}$ for certain positive value of λ. These functionals can be seen as generalizations of the Fekete-Szegő functional $\lambda a_2^2 - a_3$. Several authors including [1, 3, 5, 10, 11, 13, 14, 15, 16, 18, 23] have investigated the Fekete-Szegő and Zalcman functionals for various subclasses of univalent and multivalent functions.

Thus we motivate to obtain a sharp estimates for the Fekete-Szegő functional for functions belonging to the class $\mathscr{K}_p^s(\phi)$. Distortion, growth and covering theorems and a subordination theorem are also derived in the present investigation.

2. Fekete-Szegő inequality

In this section we assume that ϕ is an analytic function with positive real part in $\mathbb{U}$ with $\phi(0) = 1$, $\phi'(0) > 0$ and which maps the open unit disc $\mathbb{U}$ onto a region starlike with respect to 1 which is symmetric with respect to the real axis. In such case the function ϕ has an expansion of the form

$$\phi(z) = 1 + B_1 z + B_2 z^2 + \cdots \qquad (B_1 > 0, \ z \in \mathbb{U}). \tag{7}$$

Let Ω be the class of analytic functions of the form

$$w(z) = w_1 z + w_2 z^2 + \cdots \qquad (z \in \mathbb{U}) \tag{8}$$

satisfying the condition $|w(z)| < 1$ in $\mathbb{U}$. We need the following Lemmas to prove our results:

LEMMA 1. [9] *If $w \in \Omega$, then for any complex number ν:*

$$|w_1| \leqslant 1, \qquad |w_2 - \nu w_1^2| \leqslant \max\{1, |\nu|\}.$$

The result is sharp for the functions $w(z) = z$ or $w(z) = z^2$.

LEMMA 2. [4] *Let* $f \in \mathscr{A}_p$ *of the form* (1) *belonging to the class* $\mathscr{S}_p^*(p/2)$. *Then*

$$\left|a_{p+2} - \nu a_{p+1}^2\right| \leqslant \frac{p}{2}.\max\{1, |1 + p(1-2\nu)|\} \quad (\nu \in \mathbb{C}).$$

The result is sharp.

THEOREM 1. *Let* $f \in \mathscr{A}_p$ *of the form* (1) *belonging to the class* $\mathscr{K}_p^s(\phi)$, *then*

$$|a_{p+1}| \leqslant \frac{p}{p+1} B_1 \tag{9}$$

and

$$\left|a_{p+2} - \nu a_{p+1}^2\right| \leqslant \frac{p^2}{p+2} + \frac{pB_1}{p+2}.\max\left\{1, \left|\frac{B_2}{B_1} - \frac{\nu p(p+2)}{(p+1)^2} B_1\right|\right\} \quad (\nu \in \mathbb{C}). \tag{10}$$

The results are sharp.

Proof. Let $f \in \mathscr{K}_p^s(\phi)$. In view of Definition1, there exists a Schwarz function w such that

$$\frac{1}{p}\left(\frac{(-1)^p z^{p+1} f'(z)}{g(z)g(-z)}\right) = \phi(w(z)) \qquad (z \in \mathbb{U}) \tag{11}$$

for some function $g \in \mathscr{S}_p^*(p/2)$. Let

$$g(z) = z^p + b_{p+1} z^{p+1} + b_{p+2} z^{p+2} + \cdots.$$

Then by a simple calculation, we have

$$\frac{g(z)g(-z)}{(-z)^p} = z^p + (2b_{p+2} - b_{p+1}^2) z^{p+2} + \cdots,$$

so that,

$$\frac{(-z)^p}{g(z)g(-z)} = \frac{1}{z^p} - (2b_{p+2} - b_{p+1}^2)\frac{1}{z^{p-2}} + \cdots. \tag{12}$$

Series expansion (12) and Taylor expansion (1) for f, give

$$\frac{1}{p}\left(\frac{(-1)^p z^{p+1} f'(z)}{g(z)g(-z)}\right) = 1 + \frac{p+1}{p} a_{p+1} z + \left(\frac{p+2}{p} a_{p+2} - 2b_{p+2} + b_{p+1}^2\right) z^2 + \cdots. \tag{13}$$

Also,

$$\phi(w(z)) = 1 + B_1 w_1 z + (B_1 w_2 + B_2 w_1^2) z^2 + \cdots. \tag{14}$$

Making use of (13), (14) in (11) and then equating the coefficients of z and z^2 in the resulting equation, we get

$$a_{p+1} = \frac{p}{p+1} B_1 w_1$$

and

$$a_{p+2} = \frac{p}{p+2}\left(2b_{p+2} - b_{p+1}^2 + B_1 w_2 + B_2 w_1^2\right).$$

Thus for a complex number ν, we have

$$a_{p+2} - \nu a_{p+1}^2 = \frac{p}{p+2}\left(2b_{p+2} - b_{p+1}^2 + B_1 w_2 + B_2 w_1^2\right) - \nu\left(\frac{p}{p+1}B_1 w_1\right)^2$$

$$\begin{aligned}|a_{p+2} - \nu a_{p+1}^2| &= \frac{2p}{p+2}\left|\left(b_{p+2} - \frac{1}{2}b_{p+1}^2\right) + \frac{B_1}{2}\left\{w_2 - \left(\frac{\nu p(p+2)B_1}{(p+1)^2} - \frac{B_2}{B_1}\right)w_1^2\right\}\right| \\ &\leqslant \frac{2p}{p+2}\left|b_{p+2} - \frac{1}{2}b_{p+1}^2\right| + \frac{pB_1}{p+2}\left|w_2 - \left(\frac{\nu p(p+2)B_1}{(p+1)^2} - \frac{B_2}{B_1}\right)w_1^2\right|.\end{aligned}$$

By virtue of Lemma 1 and Lemma 2, we have

$$|a_{p+2} - \nu a_{p+1}^2| \leqslant \frac{p^2}{p+2} + \frac{pB_1}{p+2}.\max\left\{1, \left|\frac{\nu p(p+2)}{(p+1)^2}B_1 - \frac{B_2}{B_1}\right|\right\}.$$

This completes the required assertions (9) and (10).

For sharpness consider the function f_1 by

$$f_1(z) = p\int_0^z \frac{u^{p-1}}{(1-u^2)^p}\phi(u)du\,.$$

The function f_1 clearly belongs to the class $\mathscr{K}_p^s(\phi)$ with $g(z) = \frac{z^p}{(1-z)^p} \in \mathscr{S}_p^*(p/2)$. Since

$$\frac{pz^{p-1}}{(1-z^2)^p}\phi(z) = p\{z^{p-1} + B_1 z^p + (B_2+p)z^{p+1} + \cdots\},$$

we have

$$\begin{aligned}f_1(z) &= p\int_0^z \{u^{p-1} + B_1 u^p + (B_2+p)u^{p+1} + \cdots\}du \\ &= z^p + \frac{pB_1}{p+1}z^{p+1} + \frac{p(B_2+p)}{p+2}z^{p+2} + \cdots.\end{aligned}$$

Next, we consider

$$f_2(z) = p\int_0^z \frac{u^{p-1}}{(1-u^2)^p}\phi(u^2)du.$$

Then, we obtain

$$f_2(z) = z^p + \frac{p(B_1+p)}{p+2}z^{p+2} + \cdots.$$

Functions f_1 and f_2 show that the results (9) and (10) are sharp. □

REMARK 1. Letting $p = 1$ in Theorem 1, we have [[6], Theorem 1].

3. Distortion and growth Theorems

THEOREM 2. *Let ϕ be an analytic univalent function with positive real part and*

$$\phi(-r) = \min_{|z|=r<1} |\phi(z)|, \qquad \phi(r) = \max_{|z|=r<1} |\phi(z)|.$$

If p is an odd number and f belongs to the class $\mathscr{K}_p^s(\phi)$, then

$$\frac{\phi(-r)r^{p-1}}{(1+r^2)^p} \leqslant |f'(z)| \leqslant \frac{\phi(r)r^{p-1}}{(1-r^2)^p} \quad (|z| = r < 1) \tag{15}$$

and

$$\int_0^r \frac{\phi(-l)l^{p-1}}{(1+l^2)^p} dl \leqslant |f(z)| \leqslant \int_0^r \frac{\phi(l)l^{p-1}}{(1-l^2)^p} dl \quad (|z| = r < 1). \tag{16}$$

The results are sharp.

Proof. Suppose $f \in \mathscr{K}_p^s(\phi)$. By (6), we have

$$\frac{zf'(z)}{pG(z)} \prec \phi(z) \tag{17}$$

where

$$G(z) = \frac{g(z)g(-z)}{(-z)^p}$$

is an odd p-valent starlike function, which has the inequalities

$$\frac{r^p}{(1+r^2)^p} \leqslant |G(z)| \leqslant \frac{r^p}{(1-r^2)^p} \quad (|z| = r < 1).$$

From (17) for a Schwarz funcion w, we have

$$\begin{aligned} |f'(z)| &= \frac{p|G(z)|}{|z|} |\phi(w(z))| \\ &\leqslant \frac{pr^{p-1}}{(1-r^2)^p} . \max_{|z|=r} |\phi(z)| \quad (|z| = r < 1) \\ &\leqslant \frac{pr^{p-1}}{(1-r^2)^p} \phi(r) \qquad (|z| = r < 1). \end{aligned}$$

Similarly

$$|f'(z)| \geqslant \frac{pr^{p-1}}{(1+r^2)^p} \phi(-r) \quad (|z| = r < 1).$$

To prove the sharpness of our results, we consider the functions

$$f_1(z) = p \int_0^z \frac{u^{p-1}}{(1-u^2)^p} \phi(u)\, du$$

and

$$f_2(z) = p \int_0^z \frac{u^{p-1}}{(1+u^2)^p} \phi(u)\, du.$$

Clearly f_1 and f_2 are of p-valant close to convex functions with $g_1(z) = \frac{z^p}{(1-z)^p}$ and $g_2(z) = \frac{z^p}{(1+z^2)^{\frac{p}{2}}}$ respectively. Functions g_1 and g_2 are of p-valent starlike of order $p/2$. Thus the functions f_1 and f_2 are members of the class $\mathscr{K}_p^s(\phi)$. The sharpness of upper estimates for $|f'|$ and $|f|$ are given by the function f_1 while the sharpness for lower estimates are provided by f_2. □

REMARK 2. Letting $p = 1$ in Theorem 3, we have [[6], Theorem 2].

4. Subordination Theorem

Let $f(z) = \sum a_n z^n$ and $g(z) = \sum b_n z^n$ be two analytic functions defined in $\mathbb{U}$. Then their Hadamard product (or convolution) is the function $f * g$ defined by

$$(f * g)(z) = \sum a_n b_n z^n.$$

We need the following lemma to prove our next subordination theorem:

LEMMA 3. [20] *Let h and ψ be convex in $\mathbb{U}$ and suppose $f \prec \psi$, then $f * h \prec \psi * h$.*

THEOREM 3. *If $f \in \mathscr{K}_p^s(\phi)$, then there exists $q \prec \phi$ such that for all s and t with $|s| \leqslant 1$ and $|t| \leqslant 1$,*

$$\frac{t^{p-1} f'(sz) q(tz)}{s^{p-1} f'(tz) q(sz)} \prec \left(\frac{1-tz}{1-sz}\right)^{2p} \qquad (z \in \mathbb{U}). \tag{18}$$

Proof. By the definition of $f \in \mathscr{K}_p^s(\phi)$, there exist functions g and q such that $g \in \mathscr{S}_p^*(p/2)$, $q(z) \prec \phi(z)$ and

$$\frac{(-1)^p z^{p+1} f'(z)}{p g(z) g(-z)} = q(z). \tag{19}$$

Put $G(z) = \frac{g(z)g(-z)}{(-z)^p}$ in (19). Then, we have $\frac{zf'(z)}{G(z)} = q(z)$, which implies that

$$\frac{zf''(z)}{f'(z)} - \frac{zq'(z)}{q(z)} + 1 - p = \frac{zG'(z)}{G(z)} - p\,. \tag{20}$$

Since, $G \in \mathscr{S}_p^*$, we have

$$\frac{zG'(z)}{pG(z)} \prec \frac{1+z}{1-z}$$

and hence

$$\frac{zG'(z)}{G(z)} - p \prec \frac{2pz}{1-z}. \tag{21}$$

For s and t such that $|s| \leqslant 1$, $t \leqslant 1$, the function

$$h(z) = \int_0^z \Big(\frac{s}{1-su} - \frac{t}{1-tu}\Big) du \tag{22}$$

is convex in $\mathbb{U}$. In view of (20), (21) and (22) applying Lemma 3, we have

$$\Big(\frac{zf''(z)}{f'(z)} - \frac{zq'(z)}{q(z)} + 1 - p\Big) * h(z) \prec \frac{2pz}{1-z} * h(z). \tag{23}$$

Given any function k analytic in $\mathbb{U}$, with $k(0) = 0$, we have

$$(k*h)(z) = \int_{tz}^{sz} k(u)\frac{du}{u} \qquad (z \in \mathbb{U}). \tag{24}$$

From (23) and (24), we get

$$\int_{tz}^{sz} \Big(\frac{uf''(u)}{f'(u)} - \frac{ug'(u)}{g(u)} + 1 - p\Big)\frac{du}{u} \prec \int_{tz}^{sz} \frac{2p}{1-u} du\,,$$

which implies that

$$\frac{(sz)^{1-p}f'(sz)q(tz)}{(tz)^{1-p}f'(tz)q(sz)} \prec \Big[\frac{1-tz}{1-sz}\Big]^{2p}.$$

This completes the proof of the theorem. $\square$

Acknowledgements. The author thanks to referee to their valuable suggestions which led to the improvement of this paper.

Conflicts of interest. The author declare that there is no conflict of interest regarding the publication of this paper.

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(Received April 7, 2018)

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