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NONLINEAR DYNAMIC EQUATIONS ON TIME SCALES WITH IMPULSES AND NONLOCAL CONDITIONS

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Abstract. The purpose of this paper is to introduce more general results on the existence of solutions for nonlinear dynamic equations on time scales with impulses and nonlocal initial conditions. We establish the existence of solutions by applying a fixed point result due to O'Regan, while the uniqueness of solutions is obtained through the contraction mapping principle. Our results extend previous work in the literature and an example is discussed to illustrate the obtained results.

1. Introduction

In [28], Hilger introduced the theory of time scales, which unifies the discrete and continuous analysis, and the field also includes quantum calculus as a special case. This theory enables the researchers to study both difference and differential equations under one framework, called dynamic equations on time scales. Dynamic equations on time scales are applicable to either discrete or continuous models, and to those so-called hybrid models that combine discrete and continuous cases. We refer the reader to [1, 11, 12, 13, 14, 16, 38, 39] and references therein for several studies in the context of theory of time scales.

The theory of impulsive dynamic equations provide an excellent tool for the mathematical modelling of various real-world phenomena that involve abrupt changes at certain moments during their evolution; for example, natural disasters, certain diseases, industrial robotics, etc. In particular, work related to impulsive dynamic equations can be observed, see [4, 9, 21, 24, 29, 30, 33]. In the last fifteen years, several researchers and authors have focused their attention to the theory of impulsive dynamic equations on time scales, covering a variety of different problems, for instance, see [5, 15, 22, 23, 26, 27, 40]. This is mainly because of the rich theory of impulsive differential equations, for instance, see, [8, 10, 31, 36, 41] and the applicability of dynamic equations on time scales in various branches of science and engineering, among others, in control system [34], in population dynamics [42], and even in economics theory [6, 7], to mention the few.

However, as per our knowledge, not much has been developed in the direction of impulsive dynamic equations with nonlocal conditions. The mathematical modelling

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of certain phenomena found in physics, biology, control theory, and engineering leads to the study of nonlocal problems. As a matter of fact, in various situations, equations coupled with the nonlocal condition are found to be more advantageous than those with the traditional local condition. Due to having a wide range of applications, the study of nonlocal problems is treated as a very interesting and important field. This can be witnessed by numerous significant works available in the literature, see [3, 17, 18, 19, 20, 22].

In [21], Chang and Li used Sadovskii's fixed point theorem to establish the existence theorems for the impulsive dynamic equations of the type

$$\begin{aligned} u^{\Delta}(t)+p(t)u^{\sigma}(t) &= f\big(t,u(t)\big), \ \text{a.e. } t\in[0,T]_{\mathbb{T}},\ t\neq t_k,\ k=1,2,\dots,m;\\ u(t_k^{+})-u(t_k^{-}) &= \mathscr{I}_k\big(u(t_k)\big),\ k=1,2,\dots,m;\\ u(0) &= \Phi(u), \end{aligned} \tag{1}$$

where $u(t_k^{\pm})=\lim_{t\to t_k^{\pm}}u(t)$, $f:\mathbb{T}\times\mathbb{R}\to\mathbb{R}$ and $\mathscr{I}_k:\mathbb{R}\to\mathbb{R}$ are continuous, and $\Phi:\mathscr{C}\big([0,T]_{\mathbb{T}}\setminus\{t_1,t_2,\dots,t_m\},\mathbb{R}\big)\to\mathbb{R}$ is a given function. The idea in [21] was to extend the results of [9] to the nonlocal initial value problem (1) by dropping the boundedness of impulse functions.

Quite recently, in [4], Ardjouni and Djouni have employed a modification of Krasnoselskii's fixed point theorem due to Burton and presented the existence result for solutions to the nonlinear impulsive dynamic equations of the type

$$\begin{aligned} u^{\Delta}(t)+p(t)\lambda\big(u^{\sigma}(t)\big) &= f\big(t,u(t)\big),\ \ t\in(0,T]_{\mathbb{T}};\\ u(t_k^{+})-u(t_k^{-}) &= \mathscr{I}_k\big(t_k,u(t_k)\big),\ k=1,2,\dots,m;\\ u(0) &= 0, \end{aligned} \tag{2}$$

where $\lambda:\mathbb{R}\to\mathbb{R}$ is a continuous function and $f:\mathbb{T}\times\mathbb{R}\to\mathbb{R}$ and $\mathscr{I}_k:\mathbb{T}\times\mathbb{R}\to\mathbb{R}$ satisfying a Lipschitz condition. Here we notice that equation (2) is totally nonlinear. Therefore the solutions of dynamic problem (2) cannot be expressed in the explicit form. Hence the method of variation of parameters is not applicable.

Motivated by the work of the above mentioned papers and [21], [4], in the present paper, we investigate the existence of solutions to the following type of nonlinear dynamic equations on time scales with impulses and nonlocal initial condition

$$\begin{aligned} u^{\Delta}(t)+p(t)\lambda\big(u^{\sigma}(t)\big) &= f\big(t,u(t)\big),\ \ t\in\mathbb{I}^{\kappa},\ t\neq t_k,\ k=1,2,\dots,m;\\ u(t_k^{+})-u(t_k^{-}) &= \mathscr{I}_k\big(u(t_k)\big),\ k=1,2,\dots,m;\\ u(0) &= \Phi(u), \end{aligned} \tag{3}$$

where $\mathbb{I}:=[0,T]\cap\mathbb{T}$, $T\in\mathbb{T}$ with $T>0$, $\mathbb{T}$ is a time scale containing at least finitely-many right-dense points, $p:\mathbb{T}\to\mathbb{R}$ is regressive and rd-continuous, $\lambda:\mathbb{R}\to\mathbb{R}$ is continuous, $f:\mathbb{T}\times\mathbb{R}\to\mathbb{R}$ is rd-continuous, and $\Phi:\mathscr{C}\big(\mathbb{I}\setminus\{t_1,t_2,\dots,t_m\},\mathbb{R}\big)\to\mathbb{R}$ is a given function, possibly nonlinear. We assume that $0=t_0<t_1<t_2<\dots<t_m<t_{m+1}=T$ are a priori known moments of impulse, $\{t_k\}_{k=1}^{m}\subset\mathbb{I}$, and t_k is right-dense for $k=1,2,\dots,m$. The terms $u(t_k^{+})$ and $u(t_k^{-})$ represents right and left limits of u

at $t = t_k$ in the sense of time scales. For each $k = 1,2,\dots,m$, $\mathscr{I}_k$ is a continuous real valued function defined on $\mathbb{R}$, which describes the discontinuity of u at t_k.

The existence results presented in the current investigation concerning the impulsive dynamic problem with nonlocal initial conditions extend and generalize the earlier existence results for the class of dynamic initial value problems. More precisely,

- For identity function λ and $\phi = 0$, the results of this paper here reduce to those of [29].
- For $\Phi = 0$, the results included in this paper reduce to those of [4].
- For identity function λ and without impulsive conditions, the results presented here are similar to those considered in [37].
- For identity function λ and constant function Φ, the results of [32] are included here as a special case.

The impulsive dynamic system (3) will provide a basic model to study the dynamics of hybrid continuous–discrete nonlocal phenomena, like a nonlocal neural network, nonlocal pollution, nonlocal combustion, that are subject to abrupt changes. To obtain the desired results, first, we reformulate the impulsive dynamic system (3) as an equivalent delta integral system and then apply O'Regan's fixed point theorem [35]. The obtained integral system is the sum of two mappings, one is completely continuous and the other is nonlinear contraction. The novelty of the present paper is considering a new type of nonlinear impulsive dynamic problem (3), then presenting results concerning the existence of at least one solution using fixed point theorem and finding reasonable condition for the uniqueness of solutions.

The set up of the paper is as follows. In Section 2, we recall some notions of time scales calculus and results from fixed point theory that are required to achieve the main results. In Section 3, we present our main results concerning the existence of at least one solution and the uniqueness of solutions of (3). Finally, in Section 4, we provide simple examples illustrating the obtained results.

2. Preliminary

In this section, we recall several definitions and some results which help the reader to follow the paper easily. The following basic knowledge of the theory of time scales is taken from [12, 13].

A time scale is an arbitrary nonempty, closed subset of the real numbers $\mathbb{R}$. It is denoted by $\mathbb{T}$. The forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$, at the point $t \in \mathbb{T}$ is defined by $\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}$ and, similarly, the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$ is $\rho(t) := \sup\{s \in \mathbb{T} : s < t\}$. We make the convention that $\inf\phi = \sup\mathbb{T}$ and $\sup\phi = \inf\mathbb{T}$. The graininess function $\mu : \mathbb{T} \to [0,\infty)$ is defined by $\mu(t) := \sigma(t) - t$.

A point $t \in \mathbb{T}$ is said to be right-scattered if $\sigma(t) > t$; while it is left-scattered if $\rho(t) < t$. If $t < \sup\mathbb{T}$ and $\sigma(t) = t$, then we say t is right-dense; while t is left-dense if $t > \inf\mathbb{T}$ and $\rho(t) = t$.

If m is a possible left-scattered maximum, then we write $\mathbb{T}^{\kappa} := \mathbb{T} \setminus \{m\}$. Otherwise $\mathbb{T}^{\kappa} := \mathbb{T}$.

DEFINITION 1. A function $u : \mathbb{T} \to \mathbb{R}$ is said to be delta differentiable at $t \in \mathbb{T}^{\kappa}$ if there exists a number $u^{\Delta}(t) \in \mathbb{R}$ such that for given $\varepsilon > 0$ there is a neighbourhood N of t with

$$\left|u\big(\sigma(t)\big) - u(s) - u^{\Delta}(t)\big(\sigma(t) - s\big)\right| \leqslant \varepsilon\left|\sigma(t) - s\right|, \text{ for all } s \in N.$$

The number $u^{\Delta}(t)$ is known as the delta derivative of u at t. For brevity, we write $u^{\sigma} := u \circ \sigma$.

DEFINITION 2. A function $u : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at every right-dense points in $\mathbb{T}$ and its left sided limits exist at left-dense points in $\mathbb{T}$. The notation for the set of all rd-continuous functions with domain $\mathbb{T}$ and taking values in $\mathbb{R}$ is $\mathscr{C}_{rd}(\mathbb{T}, \mathbb{R})$.

DEFINITION 3. A function $f : \mathbb{T} \times \mathbb{R} \to \mathbb{R}$ is said to be rd-continuous on $\mathbb{T} \times \mathbb{R}$ if $f(\cdot, x)$ is rd-continuous on $\mathbb{T}$ for each fixed $x \in \mathbb{R}$ and $f(t, \cdot)$ is continuous on $\mathbb{R}$ for each fixed $t \in \mathbb{T}$. The notation for the set of all rd-continuous functions with domain $\mathbb{T} \times \mathbb{R}$ and taking values in $\mathbb{R}$ is $\mathscr{C}_{rd}(\mathbb{T} \times \mathbb{R}, \mathbb{R})$.

DEFINITION 4. A function $p : \mathbb{T} \to \mathbb{R}$ is said to be regressive if $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^{\kappa}$. The notation for the set of all regressive functions with domain $\mathbb{T}$ and taking values in $\mathbb{R}$ is $\mathscr{R}(\mathbb{T}, \mathbb{R})$.

Converse to the delta derivative, we can define the delta integral as follows.

DEFINITION 5. Let $u \in \mathscr{C}_{rd}(\mathbb{T}, \mathbb{R})$. If $U^{\Delta}(t) = u(t)$ for each $t \in \mathbb{T}^{\kappa}$, then the delta integral of u is defined by

$$\int_a^t u(s)\Delta s = U(t) - U(a), \text{ where } a \in \mathbb{T}.$$

REMARK 1. If u is delta differentiable at $t \in \mathbb{T}^{\kappa}$, then u is rd-continuous at $t \in \mathbb{T}^{\kappa}$.

DEFINITION 6. For $p \in \mathscr{R}(\mathbb{T}, \mathbb{R})$, the exponential function $e_p(\cdot, t_0)$ on the time scale $\mathbb{T}$ is defined as the unique solution of the initial value dynamic problem

$$u^{\Delta}(t) = p(t)u, \; u(t_0) = 1, \; t, t_0 \in \mathbb{T}.$$

For $p, q \in \mathscr{R}(\mathbb{T}, \mathbb{R})$, we define the following.

$$p \oplus q := p + q + \mu pq, \; \ominus p := \frac{-p}{1 + \mu p}, \; p \ominus q := p \oplus (\ominus q).$$

In this paper, we denote $E_0 := \sup\limits_{t \in \mathbb{I}} \left|e_{\ominus p}(t, 0)\right|$ and $E := \sup\limits_{s,t \in \mathbb{I}} \left|e_{\ominus p}(t, s)\right|$.

THEOREM 1. *For* $p,q \in \mathscr{R}(\mathbb{T},\mathbb{R}) \cap \mathscr{C}_{rd}(\mathbb{T},\mathbb{R})$. *The following hold.*

(i) $e_0(t,s) \equiv 1$ *and* $e_p(t,t) \equiv 1$;

(ii) $e_p(\sigma(t),s) = \big(1+\mu(t)p(t)\big)e_p(t,s)$;

(iii) $1/e_p(t,s) = e_{\ominus p}(t,s)$;

(iv) $e_p(t,s) = 1/e_p(s,t)$;

(v) $e_p(t,s)e_p(s,r) = e_p(t,r)$;

(vi) $e_p(t,s)e_q(t,s) = e_{p\oplus q}(t,s)$;

(vii) $e_p(t,s)/e_q(t,s) = e_{p\ominus q}(t,s)$.

Let $\mathscr{C}(\mathbb{I},\mathbb{R})$ be the space of all continuous functions with domain $\mathbb{I}$ and taking values in $\mathbb{R}$. We write $J_0 = [0,t_1]$ and for each $k = 1,2,\ldots,m$, $J_k = (t_k,t_{k+1}]$.

Define

$$\mathscr{PC} = \big\{u : \mathbb{I} \to \mathbb{R} : u \in \mathscr{C}(J_k,\mathbb{R}), \text{ and } u(t_k^+),\ u(t_k^-) \text{ exist with } u(t_k^-) = u(t_k), k = 1,2,\ldots,m\big\}$$

and

$$\mathscr{PC}^1 = \big\{u : \mathbb{I} \to \mathbb{R} : u \in \mathscr{C}^1_{rd}(J_k,\mathbb{R}), k = 1,2,\ldots,m\big\},$$

where $\mathscr{C}^1_{rd}(J_k,\mathbb{R})$ is the space of all rd-continuously delta differentiable functions with domain J_k and taking values in $\mathbb{R}$.

The set $\mathscr{PC}$ is a Banach space coupled with the norm $\|u\|_{\mathscr{PC}} := \max\limits_{1\leqslant k\leqslant m}\big\{\|u\|_k\big\}$, where $\|u\|_k = \sup\limits_{t\in J_k}\big|u(t)\big|$.

DEFINITION 7. A function $u \in \mathscr{PC}^1$ is said to be a solution of the impulsive dynamic problem (3), if u satisfies the dynamic equation $u^{\Delta}(t) + p(t)\lambda\big(u^{\sigma}(t)\big) = f\big(t,u(t)\big)$ everywhere on $\mathbb{I}^{\kappa}\setminus\{t_k\}$, $k = 1,2,\ldots,m$, and the conditions $u(t_k^+) - u(t_k^-) = \mathscr{I}_k\big(u(t_k)\big)$, $k = 1,2,\ldots,m$; $u(0) = \Phi(u)$,.

DEFINITION 8. [25] Let X, Y be two Banach spaces. A mapping $F : X \to Y$ is said to be completely continuous if the image of each bounded set B of X, $F(B)$, is relatively compact in Y.

DEFINITION 9. [35] Let X be a Banach space and $F : X \to X$. Then F is said to be nonlinear contraction map if there exists a continuous nondecreasing function $\Omega : [0,\infty) \to [0,\infty)$ satisfying $\Omega(z) < z$ for $z > 0$ such that

$$\big\|F(x) - F(y)\big\|_X \leqslant \Omega\big(\|x-y\|_X\big)$$

for all $x,y \in X$.

The time scales version of the Arzela–Ascoli theorem stated as follows.

THEOREM 2. [2] *A subset* $\mathscr{D}$ *of* $\mathscr{C}(\mathbb{I},\mathbb{R})$ *is relatively compact if and only if it is bounded and equicontinuous.*

For the existence of at least one solution of the impulsive dynamic problem (3), we shall rely on the following extension of Krasnoselskii' fixed point theorem due to O'Regan.

THEOREM 3. [35] *Let* B *be an open set in a closed, convex subset* C *of a Banach space* X. *Assume* $0 \in B$, $F(\overline{B})$ *bounded and* $F : \overline{B} \to C$ *is given by* $F := F_1 + F_2$, *where* $F_1 : \overline{B} \to X$ *is continuous and completely continuous and* $F_2 : \overline{B} \to X$ *is nonlinear contraction. Then either,*

(A1) F *has a fixed point in* $\overline{B}$; *or*

(A2) there is a point $x \in \partial B$ *and* $\xi \in (0,1)$ *with* $x = \xi F(x)$.

3. Main results

We shall prove our existence result as an application of Theorem 3. For this, first we define F, F_1, F_2 as follows.

Let $B_r := \left\{u \in \mathscr{PC}^1 : \|u\|_{\mathscr{PC}} \leqslant r\right\}$. The mapping $F : B_r \to \mathscr{PC}$ is given by

$$[Fu](t) := [F_1u](t) + [F_2u](t), \quad t \in \mathbb{I};$$

where $F_i : B_r \to \mathscr{PC}$ $(i = 1,2)$ are given by

$$[F_1u](t) := e_{\ominus p}(t,0)\Phi(u) + \int_0^t e_{\ominus p}(t,s) f\big(s,u(s)\big)\Delta s \tag{4}$$

and

$$[F_2u](t) := \int_0^t e_{\ominus p}(t,s)p(s)\Lambda\big(u(s)\big)\Delta s + \sum_{0<t_k<t} e_{\ominus p}(t,t_k)\mathscr{I}_k\big(u(t_k)\big), \tag{5}$$

where $\Lambda\big(u(s)\big) := u^{\sigma}(s) - \lambda\left(u^{\sigma}(s)\right)$. Then, we set the following notations:

$$\beta := E\ \Lambda^*\ P + \eta + r \sup_{t\in\mathscr{T}} \sum_{k=1}^{m} \left|e_{\ominus p}(t,t_k)\right| d_k < \infty$$

and

$$\gamma := E\ \alpha\ P + \sup_{t\in\mathscr{T}} \sum_{k=1}^{m} \left|e_{\ominus p}(t,t_k)\right| d_k < 1,$$

where d_k is some positive constant;

$$\Lambda^* := \max\left\{\left|\Lambda(-r)\right|, \left|\Lambda(r)\right|\right\};$$

$$P := \int_0^T \left|p(s)\right|\Delta s < \infty;$$

$$\eta := \sup_{t\in\mathscr{T}} \sum_{k=1}^{m} \left|e_{\ominus p}(t,t_k)\right|\left|\mathscr{I}_k(0)\right| < \infty;$$

$$\alpha := \left|1 - \inf_{t\in(-r,r)} \lambda'(t)\right|.$$

Next, we make the list of hypotheses that are needed to prove our main results.

(H1) The function $f : \mathbb{I}\times\mathbb{R} \to \mathbb{R}$ is rd-continuous.

(H2) There exist a continuous nondecreasing function $\psi : [0,\infty) \to [0,\infty)$ and a function $\phi \in \mathscr{C}\left(\mathbb{I},\mathbb{R}^+\right)$ such that

$$\left|f(t,y)\right| \leqslant \phi(t)\psi(|y|)$$

for every $t\in\mathbb{I}$ and $y\in\mathbb{R}$.

(H3) There exists a nondecreasing function $\Psi : \mathbb{R}^+ \to \mathbb{R}^+$ such that

$$\left|\Phi(y)\right| \leqslant \Psi\left(|y|\right)$$

for every $y \in \mathscr{C}\left(\mathbb{I}\setminus\{t_k\}_{k=1}^m;\mathbb{R}\right)$.

(H4) There exist positive constants d_k such that

$$\left|\mathscr{I}_k\left(x(t_k)\right) - \mathscr{I}_k\left(y(t_k)\right)\right| \leqslant d_k\|x-y\|_{\mathscr{PC}},\ k = 1,2,\dots,m;$$

for every $x,y\in\mathscr{PC}$.

(H5) For every positive r,

$$\frac{r}{\beta + E_0\Psi(r) + E\psi(r)\int_0^T \phi(s)\Delta s} > 1.$$

(H6) There exists a positive constant L_1 such that

$$\left|f(t,x) - f(t,y)\right| \leqslant L_1|x-y|$$

for all $t\in\mathbb{I}$ and for all $x,y\in\mathbb{R}$.

(H7) There exists a positive constant L_2 such that

$$\left|\Phi(x) - \Phi(y)\right| \leqslant L_2|x-y|$$

for all $x,y\in\mathbb{R}$.

Below, we present an auxiliary lemma which reformulate our impulsive dynamic problem (3) as equivalent delta integral equation. The proof of this lemma parallels that of [4, Lemma 3.1] and hence omitted.

LEMMA 1. *The function $u\in\mathscr{PC}^1$ is a solution of the impulsive dynamic problem* (3) *if and only if $u\in\mathscr{PC}$ satisfy*

$$\begin{aligned} u(t) &= e_{\ominus p}(t,0)\Phi(u) + \int_0^t e_{\ominus p}(t,s)f\left(s,u(s)\right)\Delta s \\ &\quad + \int_0^t e_{\ominus p}(t,s)p(s)\Lambda\left(u(s)\right)\Delta s + \sum_{0<t_k<t} e_{\ominus p}(t,t_k)\mathscr{I}_k\left(u(t_k)\right) \end{aligned} \tag{6}$$

for all $t\in\mathbb{I}$.

Now we are in a position to present our main results. The first one is based on Theorem 3.

THEOREM 4. *Assume that the hypotheses (H1)–(H5) are satisfied. In addition, suppose that* λ *is differentiable, increasing function on* $(-r,r)$ *such that* $0 \leqslant \inf_{t\in(-r,r)} \lambda'(t) \leqslant 1$. *Then the impulsive dynamic problem* (3) *has at least one solution.*

Proof. We shall give the proof in several steps.

Step 1. The map $F_1 : B_r \to \mathscr{PC}$ is continuous and completely continuous.

Let $\{u_n\}$ be a sequence of elements of B_r converges to u in B_r. Then we see that

$$\begin{aligned}
&\big|[F_1u_n](t) - [F_1u](t)\big| \\
&= \left| e_{\ominus p}(t,0)\big[\Phi(u_n) - \Phi(u)\big] + \int_0^t e_{\ominus p}(t,s)\big[f(s,u_n(s)) - f(s,u(s))\big]\Delta s \right| \\
&\leqslant \big|e_{\ominus p}(t,0)\big|\big|\Phi(u_n) - \Phi(u)\big| + \int_0^t \big|e_{\ominus p}(t,s)\big|\big|f(s,u_n(s)) - f(s,u(s))\big|\Delta s.
\end{aligned}$$

Using the hypothesis (H2), we can write

$$\big\|F_1(u_n) - F_1(u)\big\|_{\mathscr{PC}} \leqslant E_0\big|\Phi(u_n) - \Phi(u)\big| + E\int_0^t \phi(s)\big|\psi(|u_n|) - \psi(|u|)\big|\Delta s$$

which yields, by continuities of ψ and Φ that

$$\big\|F_1(u_n) - F_1(u)\big\|_{\mathscr{PC}} \to 0 \ \text{ as } n \to \infty.$$

Thus, F_1 is uniformly continuous on B_r and hence is continuous on B_r.

Now, we show that F_1 is completely continuous. For this, by Theorem 2, it is enough to show that $F_1(B_r)$ is bounded and equicontinuous.

For any $u \in B_r$ and $t \in \mathbb{I}$, we have

$$\big|[F_1u](t)\big| \leqslant \big|e_{\ominus p}(t,0)\big|\big|\Phi(u)\big| + \int_0^t \big|e_{\ominus p}(t,s)\big|\big|f\big(s,u(s)\big)\big|\Delta s.$$

Using the hypotheses (H2) and (H3), we obtain

$$\begin{aligned}
\|F_1(u)\|_{\mathscr{PC}} &\leqslant E_0\,\Psi\big(|u|\big) + E\int_0^t \phi(s)\psi\big(|u|\big)\Delta s \\
&\leqslant E_0\,\Psi\big(r\big) + E\,\psi(r)\int_0^t \phi(s)\Delta s := K.
\end{aligned}$$

Thus, $\|F_1(u)\|_{\mathscr{PC}} \leqslant K$ for all $u \in B_r$. Hence $F_1(B_r)$ is bounded.

Next, we show that $F_1(B_r)$ is equicontinuous. To this end, let $t_1,t_2\in\mathbb{I}$ with $t_1\leqslant t_2$ and $u\in B_r$. Then we compute that

$$
\begin{aligned}
&\big|[F_1u](t_2)-[F_1u](t_1)\big|\\
&=\bigg|e_{\ominus p}(t_2,0)\Phi(u)-e_{\ominus p}(t_1,0)\Phi(u)+\int_0^{t_2}e_{\ominus p}(t_2,s)f\big(s,u(s)\big)\Delta s\\
&\quad-\int_0^{t_1}e_{\ominus p}(t_1,s)f\big(s,u(s)\big)\Delta s\bigg|\\
&\leqslant\big|e_{\ominus p}(t_2,0)-e_{\ominus p}(t_1,0)\big|\big|\Phi(u)\big|\\
&\quad+\bigg|\int_0^{t_2}e_{\ominus p}(t_2,s)f\big(s,u(s)\big)\Delta s-\int_0^{t_1}e_{\ominus p}(t_1,s)f\big(s,u(s)\big)\Delta s\bigg|\\
&=\big|e_{\ominus p}(t_2,0)-e_{\ominus p}(t_1,0)\big|\big|\Phi(u)\big|\\
&\quad+\bigg|e_{\ominus p}(t_2,0)\int_0^{t_1}e_p(s,0)f\big(s,u(s)\big)\Delta s+e_{\ominus p}(t_2,0)\int_{t_1}^{t_2}e_p(s,0)f\big(s,u(s)\big)\Delta s\\
&\quad-e_{\ominus p}(t_1,0)\int_0^{t_1}e_p(s,0)f\big(s,u(s)\big)\Delta s\bigg|\\
&\leqslant\big|e_{\ominus p}(t_2,0)-e_{\ominus p}(t_1,0)\big|\big|\Phi(u)\big|+\big|e_{\ominus p}(t_2,0)-e_{\ominus p}(t_1,0)\big|\int_0^{t_1}\big|e_p(s,0)\big|\big|f\big(s,u(s)\big)\big|\Delta s\\
&\quad+\big|e_{\ominus p}(t_2,0)\big|\int_{t_1}^{t_2}\big|e_p(s,0)\big|\big|f\big(s,u(s)\big)\big|\Delta s.
\end{aligned}
$$

Similarly for $t_1\geqslant t_2$ we obtain the same inequality.

Since $e_{\ominus p}(t,0)$ is continuous on $\mathbb{I}$, the right hand side tends to zero as $t_1-t_2\to 0$. Thus, we obtain that $F_1(B_r)$ is equicontinuous. Therefore we can conclude that F_1 is completely continuous on B_r.

Step 2. $F(B_r)$ is bounded.

For any $u\in B_r$ and for each $t\in\mathbb{I}$, we see that

$$
\begin{aligned}
\big|[F_2u](t)\big|&\leqslant\int_0^t\big|e_{\ominus p}(t,s)\big|\big|p(s)\big|\big|\Lambda\big(u(s)\big)\big|\Delta s+\sum_{0<t_k<t}\big|e_{\ominus p}(t,t_k)\big|\big|\mathscr{I}_k\big(u(t_k)\big)\big|\\
&\leqslant E\,\Lambda^*\int_0^t\big|p(s)\big|\Delta s+\sum_{0<t_k<t}\big|e_{\ominus p}(t,t_k)\big|\big|\mathscr{I}_k\big(u(t_k)\big)\big|\\
&\leqslant E\,\Lambda^*P+\sum_{0<t_k<t}\big|e_{\ominus p}(t,t_k)\big|\big|\mathscr{I}_k(0)\big|+\sum_{0<t_k<t}\big|e_{\ominus p}(t,t_k)\big|d_k\|u\|_{\mathscr{PC}}\\
&\leqslant E\,\Lambda^*P+\eta+r\sum_{0<t_k<t}\big|e_{\ominus p}(t,t_k)\big|d_k.
\end{aligned}
$$

Thus, in view of the hypothesis (H4), $\big\|F_2(u)\big\|_{\mathscr{PC}}\leqslant\beta$. Hence $F_2(B_r)$ is bounded. Also, from Step 1., $F_1(B_r)$ is bounded. Combining these two facts, we obtain $F(B_r)$ is bounded.

Step 3. $F_2 : B_r \to \mathscr{PC}$ is nonlinear contraction.

First, we claim that for all $u, v \in B_r$,

$$\left|\Lambda\big(u(s)\big) - \Lambda\big(v(s)\big)\right| \leqslant \alpha \|u - v\|_{\mathscr{PC}}$$

for some $\alpha \in (0,1]$. Let $u, v \in B_r$ with $u^\sigma \neq v^\sigma$. Then

$$\begin{aligned}
\left|\Lambda\big(u(s)\big) - \Lambda\big(v(s)\big)\right| &= \left|u^\sigma(s) - v^\sigma(s)\right| \left|1 - \left(\frac{\lambda\left(u^\sigma(s)\right) - \lambda\left(v^\sigma(s)\right)}{u^\sigma(s) - v^\sigma(s)}\right)\right| \\
&= \left|u^\sigma(s) - v^\sigma(s)\right| \left|1 - \lambda'(t)\right| \text{ for some } t \in (-r, r) \\
&\leqslant \left|1 - \inf_{t \in (-r,r)} \lambda'(t)\right| \left|u^\sigma(s) - v^\sigma(s)\right| \\
&= \alpha \left|u^\sigma(s) - v^\sigma(s)\right|.
\end{aligned}$$

Hence for all $u, v \in B_r$,

$$\left|\Lambda\big(u(s)\big) - \Lambda\big(v(s)\big)\right| \leqslant \alpha \|u - v\|_{\mathscr{PC}}.$$

Now for $u, v \in B_r$ and for each $t \in \mathbb{I}$, we find that

$$\begin{aligned}
\left|[F_2u](t) - [F_2v](t)\right| \leqslant &\int_0^t \left|e_{\ominus p}(t,s)\right| \left|p(s)\right| \left|\Lambda\big(u(s)\big) - \Lambda\big(v(s)\big)\right| \Delta s \\
&+ \sum_{0 < t_k < t} \left|e_{\ominus p}(t, t_k)\right| \left|\mathscr{I}_k\big(u(t_k)\big) - \mathscr{I}_k\big(v(t_k)\big)\right|
\end{aligned}$$

and using hypothesis (H4), we write

$$\begin{aligned}
\left|[F_2u](t) - [F_2v](t)\right| &\leqslant \int_0^t \left|e_{\ominus p}(t,s)\right| \left|p(s)\right| \alpha \left|u(s) - v(s)\right| \Delta s \\
&\quad + \sum_{0 < t_k < t} \left|e_{\ominus p}(t, t_k)\right| d_k \|u - v\|_{PC} \\
&\leqslant \left(E\, \alpha \int_0^t \left|p(s)\right| \Delta s + \sum_{0 < t_k < t} \left|e_{\ominus p}(t, t_k)\right| d_k\right) \|u - v\|_{\mathscr{PC}} \\
&\leqslant \gamma \|u - v\|_{\mathscr{PC}}.
\end{aligned}$$

Thus, $\left\|F_2(u) - F_2(v)\right\|_{\mathscr{PC}} \leqslant \gamma \|u - v\|_{\mathscr{PC}}$. That is, $\left\|F_2(u) - F_2(v)\right\|_{\mathscr{PC}} \leqslant \Omega\big(\|u - v\|_{\mathscr{PC}}\big)$, where $\Omega(w) = \gamma\, w$, $0 < \gamma < 1$. Hence F_2 is nonlinear contraction.

Step 4. (A2) of Theorem 3 does not occur.

To this end, we perform the argument by contradiction. Suppose that (A2) of Theorem 3. Then there is $w \in \partial B_r$ and $\xi \in (0,1)$ such that $w = \xi F(w)$. That is, for each $t \in \mathbb{I}$,

$$w(t) = \xi [F_1 w](t) + \xi [F_2 w](t).$$

We compute that

$$
\begin{aligned}
\left|w(t)\right| &\leqslant \left|[Fw](t)\right| \\
&= \Big| e_{\ominus p}(t,0)\Phi(w) + \int_0^t e_{\ominus p}(t,s) f\big(s,w(s)\big)\Delta s + \int_0^t e_{\ominus p}(t,s)p(s)\Lambda\big(w(s)\big)\Delta s \\
&\quad + \sum_{0<t_k<t} e_{\ominus p}(t,t_k)\mathscr{I}_k\big(w(t_k)\big)\Big| \\
&\leqslant \left|e_{\ominus p}(t,0)\right|\left|\Phi(w)\right| + \int_0^t \left|e_{\ominus p}(t,s)\right|\left|f\big(s,w(s)\big)\right|\Delta s \\
&\quad + \int_0^t \left|e_{\ominus p}(t,s)\right|\left|p(s)\right|\left|\Lambda\big(w(s)\big)\right|\Delta s + \sum_{0<t_k<t}\left|e_{\ominus p}(t,t_k)\right|\left|\mathscr{I}_k\big(w(t_k)\big)\right|.
\end{aligned}
$$

Using the hypotheses (H2) and (H3), we obtain

$$
\begin{aligned}
\left|w(t)\right| &\leqslant E_0\Psi\big(|w|\big) + E\psi\big(|w|\big)\int_0^t \phi(s)\Delta s + E\Lambda^*\int_0^t \left|p(s)\right|\Delta s \\
&\quad + \sum_{0<t_k<t}\left|e_{\ominus p}(t,t_k)\right|\left|\mathscr{I}_k\big(w(t_k)\big)\right| \\
&\leqslant E_0\Psi(r) + E\psi(r)\int_0^t \phi(s)\Delta s + E\Lambda^*P + \sum_{0<t_k<t}\left|e_{\ominus p}(t,t_k)\right|\left|\mathscr{I}_k\big(w(t_k)\big)\right| \\
&\leqslant E_0\Psi(r) + E\psi(r)\int_0^t \phi(s)\Delta s + E\Lambda^*P \\
&\quad + \sum_{0<t_k<t}\left|e_{\ominus p}(t,t_k)\right|\left|\mathscr{I}_k\big(w(t_k) - \mathscr{I}_k(0)\big)\right| + \sum_{0<t_k<t}\left|e_{\ominus p}(t,t_k)\right|\left|\mathscr{I}_k(0)\right|.
\end{aligned}
$$

Now, the hypothesis (H4) yields

$$
\begin{aligned}
\left|w(t)\right| &\leqslant E_0\Psi(r) + E\psi(r)\int_0^t \phi(s)\Delta s + E\Lambda^*P + \eta + \sum_{0<t_k<t}\left|e_{\ominus p}(t,t_k)\right| d_k \|w\|_{\mathscr{PC}} \\
&= E_0\Psi(r) + E\psi(r)\int_0^t \phi(s)\Delta s + E\Lambda^*P + \eta + r\sum_{0<t_k<t}\left|e_{\ominus p}(t,t_k)\right| d_k \\
&\leqslant E_0\Psi(r) + E\psi(r)\int_0^t \phi(s)\Delta s + \beta.
\end{aligned}
$$

Since $w \in \partial B_r$, we obtain

$$
r \leqslant E_0\Psi(r) + E\psi(r)\int_0^t \phi(s)\Delta s + \beta.
$$

However, it contradicts the hypothesis (H5). Hence, (A2) of Theorem 3 does not occur and therefore the map F has at least one fixed point in B_r. This completes the proof. □

REMARK 2. The conditions in the hypotheses (H2) and (H3) are global conditions on f and Φ. The Theorem 4 also holds even if the hypotheses (H2) and (H3) are replaced by the following local conditions.

(H2–L) There exist a continuous nondecreasing function $\psi : [0,\infty) \to [0,\infty)$ and a function $\phi \in \mathscr{C}(\mathscr{T},\mathbb{R}^+)$ such that

$$\left|f(t,y)\right| \leqslant \phi(t)\psi(|y|)$$

for every $t \in \mathbb{I}$ and for every $y \in B_r$.

(H3–L) There exists a nondecreasing function $\Psi : \mathbb{R}^+ \to \mathbb{R}^+$ such that

$$\left|\Phi(y)\right| \leqslant \Psi\big(|y|\big)$$

for every $y \in \mathscr{C}\big(\mathbb{I} \setminus \{t_k\}_{k=1}^m, B_r\big)$.

Next we shall make a new hypothesis and give a corollary of Theorem 4.

(H2$'$) (Sublinear growth) There exist a function $\phi \in \mathscr{C}\big(\mathbb{I},\mathbb{R}^+\big)$ and a constant $k \in (0,1]$ such that

$$\left|f(t,y)\right| \leqslant \phi(t)\ |y|^k$$

for every $t \in \mathbb{I}$ and $y \in B_r$.

COROLLARY 1. *Assume that the hypotheses (H1), (H2$'$), (H3)–(H5) are satisfied. Suppose that λ is differentiable, increasing function on $(-r,r)$ such that $0 \leqslant \inf_{t\in(-r,r)} \lambda'(t) \leqslant 1$. Then the impulsive dynamic problem* (3) *has at least one solution.*

Next, we show the uniqueness of solutions of impulsive dynamic problem (3) by using the contraction mapping principle.

THEOREM 5. *Assume that the hypotheses (H4), (H6), and (H7) are satisfied. Further, assume that λ is differentiable, increasing function on $\mathbb{R}$ such that $0 \leqslant \inf_{t\in\mathbb{R}} \lambda'(t) \leqslant 1$. Then the impulsive dynamic problem* (3) *has a unique solution in $\mathscr{PC}$ provided that $\big(E_0L_2 + EL_1T + \gamma\big) < 1$.*

Proof. We first note that if $u \in \mathscr{PC}$, then $F(u) \in \mathscr{PC}$. Now, for $u,v \in \mathscr{PC}$ and for each $t \in \mathbb{I}$, from the definition of F, we can write

$$\begin{aligned}
&\big|[Fu](t) - [Fv](t)\big| \\
&\leqslant \left|e_{\ominus p}(t,0)\right|\left|\Phi(u) - \Phi(v)\right| + \int_0^t \left|e_{\ominus p}(t,s)\right|\left|f(s,u(s)) - f(s,v(s))\right|\Delta s \\
&\quad + \int_0^t \left|e_{\ominus p}(t,s)\right|\left|p(s)\right|\left|\Lambda\big(u(s)\big) - \Lambda\big(v(s)\big)\right|\Delta s \\
&\quad + \sum_{0<t_k<t} \left|e_{\ominus p}(t,s)\right|\left|\mathscr{I}_k\big(u(t_k)\big) - \mathscr{I}_k\big(v(t_k)\big)\right|.
\end{aligned}$$

From the hypothesis (H4), (H6), and (H7), we obtain

$$\begin{aligned}
&\big|[Fu](t)-[Fv](t)\big| \\
&\leqslant E_0L_2\|u-v\|_{\mathscr{PC}}+\int_0^t \big|e_{\ominus p}(t,s)\big|L_1\|u-v\|_{\mathscr{PC}}\Delta s \\
&\quad+\int_0^t \big|e_{\ominus p}(t,s)\big|\big|p(s)\big|\alpha\|u-v\|_{\mathscr{PC}}\Delta s+\sum_{0<t_k<t}\big|e_{\ominus p}(t,s)\big|d_k\|u-v\|_{\mathscr{PC}} \\
&\leqslant \left(E_0L_2+L_1\int_0^t \big|e_{\ominus p}(t,s)\big|\Delta s+E\alpha\int_0^t\big|p(s)\big|\Delta s+\sum_{0<t_k<t}\big|e_{\ominus p}(t,s)\big|d_k\right)\|u-v\|_{\mathscr{PC}} \\
&\leqslant \left(E_0L_2+L_1ET+E\alpha P+\sum_{0<t_k<t}\big|e_{\ominus p}(t,s)\big|d_k\right)\|u-v\|_{\mathscr{PC}} \\
&\leqslant \big(E_0L_2+L_1ET+\gamma\big)\|u-v\|_{\mathscr{PC}}.
\end{aligned}$$

Hence

$$\big|[Fx](t)-[Fy](t)\big|\leqslant\big(E_0L_2+L_1ET+\gamma\big)\|u-v\|_{\mathscr{PC}}.$$

As $E_0L_2+L_1ET+\gamma<1$, the mapping F is a contraction map from $\mathscr{PC}$ into itself. Hence by contraction mapping principle, F has a unique fixed point in $\mathscr{PC}$. This completes the proof. □

4. Example

We conclude the paper with the discussion of an example.

EXAMPLE 1. Let $\mathbb{T}=[0,1]\cup[2,3]$. Consider the following impulsive dynamic problem on $\mathbb{I}=[0,3]_{\mathbb{T}}$

$$\begin{aligned}
u^{\Delta}(t)+u^{\sigma}(t)&=f\big(t,u(t)\big),\ \ t\in[0,3]_{\mathbb{T}},\ t\neq 1/2,\\
u\big((1/2)^+\big)-u\big((1/2)^-\big)&=\frac{\big|u(1/2)\big|}{2+\big|u(1/2)\big|},\\
u(0)&=\Phi(u).
\end{aligned}\tag{7}$$

Here $p=1$, λ is the identity function, and $\mathscr{I}_1\big(u(t)\big)=\dfrac{\big|u(t)\big|}{2+\big|u(t)\big|}$. Using the above data, we find that

$$\big|\mathscr{I}_1\big(u(1/2)\big)-\mathscr{I}_1\big(v(1/2)\big)\big|\leqslant\frac{1}{2}\big|u-v\big|.$$

Further, $\gamma=\sup_{t\in\mathbb{I}}\big|e_{\ominus 1}(t,1/2)\big|\frac{1}{2}<1$, $\beta=r\gamma<\infty$, $\alpha=0$, $\eta=\sup_{t\in\mathbb{I}}\big|e_{\ominus 1}(t,1/2)\big|\mathscr{I}_1(0)=0$, and $P=3$.

(a) The first example is concerned with the illustration of Theorem 4. Take

$$f\big(t,u\big)=\frac{1}{2}\left(\frac{1}{\sqrt{1+t}}\right)\big(t+|u|\big)\tag{8}$$

and

$$\Phi(u) = \frac{|u|}{1+|u|}. \tag{9}$$

We note here that $\big|f(t,u)\big| \leqslant \phi(t)\psi\big(|u|\big)$, where $\phi(t) = \left(\frac{1}{\sqrt{1+t}}\right)$ and $\psi\big(|u|\big) = \big(3+|u|\big)$. Also, $\big|\Phi(u)\big| \leqslant \Psi\big(|u|\big)$, where $\Psi\big(|u|\big) = |u|$. Thus, all the conditions of Theorem 4 with $f(t,u)$ given by (8) and $\Phi(u)$ given by (9) are satisfied. Therefore, from Theorem 4, we conclude that the impulsive dynamic problem (7) with f and Φ as defined in (8) and (9) has at least one solution.

(b) This example is concerned with the illustration of Theorem 5. We take

$$f(t,u) = \frac{1}{21}\left(u^2+5\right)^{1/2} + t \tag{10}$$

and

$$\Phi(u) = \frac{|u|}{30\big(1+|u|\big)}. \tag{11}$$

Use of the mean value theorem yields that $\big|f(t,u)-f(t,v)\big| \leqslant \frac{1}{21}|u-v|$. Hence (H6) holds with $L_1 = \frac{1}{21}$.

Also, $\big|\Phi(u)-\Phi(v)\big| \leqslant \frac{1}{30}|u-v|$. Hence (H7) holds with $L_2 = \frac{1}{30}$. Further we observe that $E_0 = 1$, $E = \frac{1}{e}$ and hence $E_0L_2 + EL_1T + \gamma < 1$. Thus, all the hypotheses of Theorem 5 holds. Therefore from Theorem 5, we conclude that the impulsive dynamic problem (7) with f and Φ as defined in (10) and (11) has unique solution.

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