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ON ABSOLUTE MATRIX SUMMABILITY METHODS

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Abstract. In this study, we have generalized Bor's theorems ([14]) for the absolute matrix summability method.

1. Introduction

A sequence (x_n) is called to be δ-quasi monotone, if $x_n \to 0$, $x_n > 0$ eventually, and $\Delta x_n \geqslant -\delta_n$, where $\Delta x_n = x_n - x_{n+1}$ and $\delta = (\delta_n)$ is a sequence of positive numbers ([1]). A sequence (λ_n) is said to be of bounded variation, denoted by $(\lambda_n) \in BV$, if $\sum_{n=1}^{\infty} |\Delta\lambda_n| < \infty$.

Let $\sum a_n$ be an infinite series with partial sums (s_n). The nth $(C,1)$ means of the sequences (s_n) and (na_n) are denoted by v_n and t_n, respectively. The series $\sum a_n$ is said to be summable $|C,1|_k$, where $k \geqslant 1$, if ([16])

$$\sum_{n=1}^{\infty} n^{k-1} |v_n - v_{n-1}|^k = \sum_{n=1}^{\infty} \frac{|t_n|^k}{n} < \infty.$$

Let (p_n) be a sequence of positive real numbers such that

$$P_n = \sum_{k=0}^{n} p_k \to \infty \quad \text{as} \quad n \to \infty, \quad (P_{-i} = p_{-i} = 0, \ i \geqslant 1).$$

The sequence-to-sequence transformation $(s_n) \to (\sigma_n)$ with

$$\sigma_n = \frac{1}{P_n} \sum_{m=0}^{n} p_m s_m$$

defines the sequence (σ_n) of the Riesz mean or,simply, $(\overline{N}, p_n)$ mean of the sequence s_n, generated by the sequence of coefficients p_n ([17]). The series $\sum a_n$ is said to be summable $|\overline{N}, p_n|_k$, $k \geqslant 1$, if ([3])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} |\sigma_n - \sigma_n|^k < \infty.$$

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If we take $p_n = 1$ for all n, then $|\overline{N}, p_n|_k$ summability is the same as $|C,1|_k$ summability ([16]). Also, if we take $k = 1$, then $|\overline{N}, p_n|_k$ summability reduces to $|\overline{N}, p_n|$ summability ([25]).

Let $A = (a_{nv})$ be a normal matrix, i.e., a lower triangular matrix with nonzero diagonal entries. Then A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \quad n = 0, 1, \dots .$$

The series $\sum a_n$ is said to be summable $|A, p_n; \delta|_k$, $k \geqslant 1$ and $\delta \geqslant 0$, if (see [21])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\delta k + k - 1} |A_n(s) - A_{n-1}(s)|^k < \infty.$$

In the special case for $\delta = 0$, the $|A, p_n; \delta|_k$ summability reduces to $|A, p_n|_k$ summability (see [24]). If we set $\delta = 0$ and $p_n = 1$ for all n, then we obtain $|A|_k$ summability (see [26]). Also if we take $a_{nv} = \frac{p_v}{P_n}$, then we have $|\overline{N}, p_n; \delta|_k$ summability (see [4]). Finally if we take $\delta = 0$ and $a_{nv} = \frac{p_v}{P_n}$, then we get $|\overline{N}, p_n|_k$ summability.

Given any sequences (u_n), (v_n), it is customary to write $v_n = O(u_n)$, if there exist η and N, for every $n > N$, $|\frac{v_n}{u_n}| \leqslant \eta$. For any matrix entry a_{nv}, we write that $\Delta_v a_{nv} = a_{nv} - a_{n,v+1}$. Now, we will introduce some necessary notations for our main theorems. Given a normal matrix $A = (a_{nv})$, we associate two lower semi-matrices $\overline{A} = (\overline{a}_{nv})$ and $\hat{A} = (\hat{a}_{nv})$ as follows:

$$\overline{a}_{nv} = \sum_{i=v}^{n} a_{ni}, \quad n, v = 0, 1, \dots$$

and

$$\hat{a}_{00} = \overline{a}_{00} = a_{00}, \quad \hat{a}_{nv} = \overline{a}_{nv} - \overline{a}_{n-1,v}, \quad n = 1, 2, \dots .$$

It may be noted that $\overline{A}$ and $\hat{A}$ are the well known matrices of series-to-sequence and series-to-series transformations, respectively. Then, we have

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v = \sum_{v=0}^{n} \overline{a}_{nv} a_v \tag{1}$$

and

$$\overline{\Delta} A_n(s) = \sum_{v=0}^{n} \hat{a}_{nv} a_v. \tag{2}$$

2. Known result

Recently, many authors have obtained some new theorems dealing with the absolute summability factors of infinite series and Fourier series. Among them, in [14], the following theorem has proved dealing with an application of δ-quasi-monotone sequences.

THEOREM 1. *Let* $X_n = \sum_{v=0}^{n} \frac{p_v}{P_v}$, *for* $n \geqslant 0$, *and let* $\lambda_n \to 0$ *as* $n \to \infty$. *Suppose that there exists a sequence of numbers* (K_n) *that is* δ*-quasi monotone with* $\sum_{n=1}^{\infty} nX_n\delta_n < \infty$, $\sum_{n=1}^{\infty} K_nX_n$ *is convergent, and* $|\Delta\lambda_n| \leqslant K_n$ *for all* n. *If conditions*

$$\sum_{n=1}^{m} \frac{p_n}{P_n} \frac{|t_n|^k}{X_n^{k-1}} = O(X_m) \quad as \quad m \to \infty,$$

$$\sum_{n=1}^{m} \frac{|t_n|^k}{nX_n^{k-1}} = O(X_m) \quad as \quad m \to \infty,$$

and

$$\sum_{n=1}^{m} \frac{P_n}{n} = O(P_m) \quad as \quad m \to \infty,$$

hold, then the series $\sum a_n\lambda_n$ *is summable* $|\overline{N}, p_n|_k$, $k \geqslant 1$.

3. Main result

There has been considerable research on absolute summability (see [2]–[14], [18]–[23], [27]–[29]). In ([14]), Bor presented a result on the $|\overline{N}, p_n|_k$ summability factors of infinite series. The aim of this paper is to generalize this theorem for the general matrix summability methods. Before we state our main result, we show $A = (a_{nv})$ is said to be of class Ω if (see [22]) A is lower triangular

$$a_{nv} \geqslant 0, \quad n, v = 0, 1, \ldots;$$

$$a_{n-1,v} \geqslant a_{nv}, \quad \text{for} \quad n \geqslant v+1,$$

$$\overline{a}_{n0} = 1, \quad n = 0, 1, \ldots .$$

Notice that A given by

$$A_1(x) = x_1 \quad \text{and} \quad A_n(x) = \frac{x_{n-1} + x_n}{2} \quad \text{for} \quad n > 1$$

is an example of a matrix of class Ω. Now, we shall prove the following theorem.

THEOREM 2. *Let* $X_n = \sum_{v=0}^{n} \frac{p_v}{P_v}$, *for* $n \geqslant 0$, *and let* $\lambda_n \to 0$ *as* $n \to \infty$. *Suppose that there exists a sequence of numbers* (K_n) *that is* δ*-quasi monotone with* $\sum_{n=1}^{\infty} nX_n\delta_n < \infty$, $\sum_{n=1}^{\infty} K_nX_n$ *is convergent, and* $|\Delta\lambda_n| \leqslant K_n$ *for all* n. *Let* $A = (a_{nv})$ *be of class* Ω. *If conditions*

$$a_{nn} = O\left(\frac{p_n}{P_n}\right),$$

$$\sum_{v=1}^{n-1} \frac{|\hat{a}_{n,v+1}|}{v} = O(a_{nn}),$$

$$\sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k} |\Delta_v(\hat{a}_{nv})| = O\left(a_{vv}\left(\frac{P_v}{p_v}\right)^{\delta k}\right) \ as \ m \to \infty,$$

$$\sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k} |\hat{a}_{n,v+1}| = O\left(\left(\frac{P_v}{p_v}\right)^{\delta k}\right) \ as \ m \to \infty,$$

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\delta k} \frac{|t_n|^k}{nX_n^{k-1}} = O(X_m) \ as \ m \to \infty,$$

and

$$\sum_{n=1}^{m} a_{nn}\left(\frac{P_n}{p_n}\right)^{\delta k} \frac{|t_n|^k}{X_n^{k-1}} = O(X_m) \ as \ m \to \infty,$$

hold, then the series $\sum a_n \lambda_n$ *is summable* $|A, p_n; \delta|_k$, $k \geqslant 1$ *and* $\delta \geqslant 0$.

Now, we need the following Lemma for the proof of our main theorem.

LEMMA 1. [2] *Under the conditions of Theorem 2.1, we have*

$$|\lambda_n| X_n = O(1) \ as \ n \to \infty, \tag{3}$$

$$nX_n|K_n| = O(1) \ as \ n \to \infty, \tag{4}$$

$$\sum_{n=1}^{\infty} nX_n|\Delta K_n| < \infty. \tag{5}$$

4. Proof of the Theorem

Let (M_n) denote A-transform of the series $\sum a_n \lambda_n$. Therefore, by (1) and (2), we write

$$\bar{\Delta} M_n = M_n - M_{n-1} = \sum_{v=1}^{n} \hat{a}_{nv} a_v \lambda_v = \sum_{v=1}^{n} \frac{\hat{a}_{nv}\lambda_v}{v} v a_v.$$

Applying Abel's transformation to $M_n - M_{n-1}$, we have

$$M_n - M_{n-1} = \sum_{v=1}^{n-1} \Delta\left(\frac{\hat{a}_{nv}\lambda_v}{v}\right) \sum_{r=1}^{v} r a_r + \frac{\hat{a}_{nn}\lambda_n}{n} \sum_{r=1}^{n} r a_r$$

$$= \sum_{v=1}^{n-1} \Delta\left(\frac{\hat{a}_{nv}\lambda_v}{v}\right)(v+1)t_v + \frac{\hat{a}_{nn}\lambda_n}{n}(n+1)t_n$$

$$= \sum_{v=1}^{n-1} \Delta_v(\hat{a}_{nv})\lambda_v t_v \frac{v+1}{v} + \sum_{v=1}^{n-1} \hat{a}_{n,v+1}\Delta\lambda_v t_v \frac{v+1}{v} + \sum_{v=1}^{n-1} \hat{a}_{n,v+1}\lambda_{v+1}\frac{t_v}{v} + a_{nn}\lambda_n t_n \frac{n+1}{n}$$

$$= M_{n,1} + M_{n,2} + M_{n,3} + M_{n,4}$$

To finish the proof of the theorem using Minkowski's inequality, it will be sufficient to show that

$$\sum_{n=1}^{\infty}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}|M_{n,r}|^k<\infty,\ \text{ for }\ r=1,2,3,4.$$

Therefore we see that

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}|M_{n,1}|^k\leqslant\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})\lambda_v t_v\frac{v+1}{v}|$$

$$=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|^{\frac{1}{k}}|\lambda_v||t_v||\Delta_v(\hat{a}_{nv})|^{\frac{k-1}{k}}\right)^k$$

$$=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})||\lambda_v|^k|t_v|^k\right)\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\right)^{k-1}$$

$$=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}a_{nn}^{k-1}\left(\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})||\lambda_v|^k|t_v|^k\right)$$

$$=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})||\lambda_v|^k|t_v|^k$$

$$=O(1)\sum_{v=1}^{m}|\lambda_v|^{k-1}|\lambda_v||t_v|^k\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}|\Delta_v(\hat{a}_{nv})|$$

$$=O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k}a_{vv}|\lambda_v|\frac{|t_v|^k}{X_v^{k-1}}.$$

Now, by applying Abel's transformation to this last sum, we obtain

$$=O(1)\sum_{v=1}^{m-1}\Delta|\lambda_v|\sum_{r=1}^{v}\left(\frac{P_r}{p_r}\right)^{\delta k}a_{rr}\frac{|t_r|^k}{X_r^{k-1}}+O(1)|\lambda_m|\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k}a_{vv}\frac{|t_v|^k}{X_v^{k-1}}$$

$$=O(1)\sum_{v=1}^{m-1}\Delta|\lambda_v|X_v+O(1)|\lambda_m|X_m$$

$$=O(1)\sum_{v=1}^{m-1}K_vX_v+O(1)|\lambda_m|X_m=O(1),\ \text{ as }\ m\to\infty$$

according to the conditions of the theorem and Lemma. From here, continuing with the Hölder's inequality in a similar way to $M_{n,1}$, the following result is obtained:

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}|M_{n,2}|^k\leqslant\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v||t_v||\frac{v+1}{v}|\right)^k$$

$$=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v||t_v|)^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left(\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}v|K_v||t_v|\right)^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left(\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}(v|K_v|)^k|t_v|^k\right)\left(\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}\right)^{k-1}$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}a_{nn}^{k-1}\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}(v|K_v|)^{k-1}v|K_v||t_v|^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|v|K_v|\frac{|t_v|^k}{vX_v^{k-1}}$$

$$= O(1)\sum_{v=1}^{m}v|K_v|\frac{|t_v|^k}{vX_v^{k-1}}\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}|\hat{a}_{n,v+1}|$$

$$= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k}\frac{|t_v|^k}{vX_v^{k-1}}v|K_v|.$$

Now, by applying Abel's transformation to this last sum, we obtain

$$= O(1)\sum_{v=1}^{m-1}\Delta(v|K_v|)\sum_{r=1}^{v}\left(\frac{P_r}{p_r}\right)^{\delta k}\frac{|t_r|^k}{rX_r^{k-1}} + O(1)m|K_m|\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k}\frac{|t_v|^k}{vX_v^{k-1}}$$

$$= O(1)\sum_{v=1}^{m-1}\Delta(v|K_v|)X_v + O(1)m|K_m|X_m$$

$$= O(1)\sum_{v=1}^{m-1}|(v+1)\Delta|K_v| - |K_v||X_v + O(1)m|K_m|X_m$$

$$= O(1)\sum_{v=1}^{m-1}v|\Delta K_v|X_v + \sum_{v=1}^{m-1}|K_v|X_v + O(1)m|K_m|X_m = O(1), \quad \text{as} \quad m\to\infty.$$

For $r=3$, we have that

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}|M_{n,3}|^k \leqslant \sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|}{v}\right)^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\lambda_{v+1}|^k\frac{|t_v|^k}{v}\right)\left(\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}\right)^{k-1}$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}a_{nn}^{k-1}\left(\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\lambda_{v+1}|^k\frac{|t_v|^k}{v}\right)$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\lambda_{v+1}|^k\frac{|t_v|^k}{v}$$

$$= O(1)\sum_{v=1}^{m}|\lambda_{v+1}|^{k-1}|\lambda_{v+1}|\frac{|t_v|^k}{v}\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}|\hat{a}_{n,v+1}|$$

$$= O(1)\sum_{v=1}^{m}|\lambda_{v+1}|\frac{|t_v|^k}{vX_v^{k-1}}\left(\frac{P_v}{p_v}\right)^{\delta k}.$$

Now applying Abel's transformation, we obtain

$$= O(1)\sum_{v=1}^{m-1}\Delta|\lambda_{v+1}|\sum_{r=1}^{v}\left(\frac{P_r}{p_r}\right)^{\delta k}\frac{|t_r|^k}{rX_r^{k-1}} + O(1)|\lambda_{m+1}|\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k}\frac{|t_v|^k}{vX_v^{k-1}}$$

$$= O(1)\sum_{v=1}^{m-1}\Delta|\lambda_v|X_v + O(1)|\lambda_{m+1}|X_{m+1}$$

$$= O(1)\sum_{v=1}^{m-1}|K_v|X_v + O(1)|\lambda_{m+1}|X_{m+1} = O(1) \text{ as } m\to\infty.$$

Finally, for $r=4$, by the similar process in $M_{n,1}$, we get

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}|M_{n,4}|^k \leqslant \sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}a_{nn}^k|\lambda_n|^k|t_n|^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}a_{nn}|\lambda_n|^{k-1}|\lambda_n||t_n|^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}a_{nn}|\lambda_n|\frac{|t_n|^k}{X_n^{k-1}} = O(1) \text{ as } m\to\infty.$$

Therefore we get that

$$\sum_{n=1}^{\infty}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}|M_{n,r}|^k < \infty, \text{ for } r = 1,2,3,4.$$

So the proof of Theorem is completed.

5. An application to trigonometric Fourier series

Let f be a periodic function with period 2π and Lebesgue integrable over $(-\pi,\pi)$. The trigonometric Fourier series of f is defined as

$$f(x) \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx) = \sum_{n=0}^{\infty}C_n(x)$$

where

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx,\ \ a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos(nx)dx,\ \ b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin(nx)dx.$$

Set $\phi(t) = \frac{1}{2}f(x+t) + f(x-t)$ and $\phi_\alpha(t) = \frac{\alpha}{t^\alpha}\int_0^t (t-u)^{\alpha-1}\phi(u)du,\ (\alpha > 0)$.

In [15], we know that if $\phi(t)$ belongs to the class $BV(0,\pi)$, then $\sigma_n(t) = O(1)$, where $\sigma_n(t)$ is Cesàro mean of the sequence $(nC_n(t))$. Therefore we write the following results.

THEOREM 3. [14] *If $\phi(t) \in BV(0,\pi)$, and the sequences (K_n), (λ_n) and (X_n) satisfy the conditions of Theorem 1, then the series $\sum C_n(t)\lambda_n$ is summable $|\overline{N}, p_n|_k$.*

THEOREM 4. *If $\phi(t) \in BV(0,\pi)$,, and the sequences (K_n), (λ_n) and (X_n) satisfy the conditions of Theorem 2, then the series $\sum C_n(t)\lambda_n$ is summable $|A, p_n; \delta|_k$.*

REFERENCES

[1] R. P. BOAS JR., *Quasi-positive sequences and trigonometric series*, Proc. Lond. Math. Soc. **14A** (1965), 38–46.

[2] H. BOR, *On quasi-monotone sequences and their applications*, Bull. Aust. Math. Soc. **43** (1991), 187–192.

[3] H. BOR, *On two summability methods*, Math. Proc. Cambridge Philos. Soc. **97** (1985), 147–149.

[4] H. BOR, *On local property of $|\overline{N}, p_n; \delta|_k$ summability of factored Fourier series*, J. Math. Anal. Appl. **179** (1993), 646–649.

[5] H. BOR, *An application of quasi-monotone sequences to infinite series and Fourier series*, Anal. Math. Phys. **8** (2018), 77–83.

[6] H. BOR, *On absolute summability of factored infinite series and trigonometric Fourier series*, Results Math. **73** (73) (2018), 116.

[7] H. BOR, *On absolute Riesz summability factors of infinite series and their application to Fourier series*, Georgian Math. J. **26** (2019), 361–366.

[8] H. BOR, *Certain new factor theorems for infinite series and trigonometric Fourier series*, Quaest. Math. **43** (2020), 441–448.

[9] H. BOR, *On an application of power increasing sequences*, Trans. A. Razmadze Math. Inst. **174** (2020), 265–269.

[10] H. BOR, *A new note on factored infinite series and trigonometric Fourier series*, C. R. Math. Acad. Sci. Paris **359** (2021), 323–328.

[11] H. BOR, *Factored infinite series and Fourier series involving almost increasing sequences*, Bull. Sci. Math. **169**, (102990) (2021).

[12] H. BOR, *A new application of a wider class of power increasing sequences*, Numer. Funct. Anal. Optim. **42** (2021), 712–720.

[13] H. BOR, *Absolute weighted arithmetic mean summability of factored infinite series and Fourier series*, Bulletin des Sciences Mathématiques **176** (2022), 103–116.

[14] H. BOR, *On certain new applications of quasi-monotone sequences*, Transactions of A. Razmadze Mathematical Institute **179** (2025), 159–164.

[15] K. K. CHEN, *Functions of bounded variation and Cesàro means of Fourier series*, Acad. Sin. Sci. Rec. **1** (1954), 283–289.

[16] T. M. FLETT, *On an extension of absolute summability and some theorems of Littlewood and Paley*, Proc. Lond. Math. Soc. **7** (1957), 113–141.

[17] G. H. HARDY, *Divergent Series*, Oxford, at the Clarendon Press, 1949.

[18] H. S. ÖZARSLAN, *Local properties of generalized absolute matrix summability of factored Fourier series*, Southeast Asian Bull. Math. **43** (2019), 263–272.

[19] H. S. ÖZARSLAN, *On the localization of factored Fourier series*, J. Comput. Anal. Appl. **29** (2021), 344–354.

[20] H. S. ÖZARSLAN, *A study on local properties of Fourier series*, Bol. Soc. Parana. Mat. (3) **39** (2021), 201–211.

[21] H. S. ÖZARSLAN AND H. N. ÖĞDÜK, *Generalization of two theorems on absolute summability methods*, Aust. J. Math. Anal. Appl. **1** (2004), 7 pp.

[22] H. N. ÖZGEN, *A note on generalized absolute summability*, An. Ştiint. Univ. Al. I. Cuza Iaşi. Mat. (N. S.), **59** (2013), 185–190.

[23] H. N. ÖZGEN, *Absolute matrix summability of factored infinite series and Fourier series*, Appl. Math. E-Notes **23** (2023), 352–359.

[24] W. T. SULAIMAN, *Inclusion theorems for absolute matrix summability methods of an infinite series (IV)*, Indian J. Pure Appl. Math. **34** (2003), 1547–1557.

[25] G. SUNOUCHI, *Notes on Fourier analysis. XVIII. Absolute summability of series with constant terms*, Tohoku Math. J. **2** 1 (1949), 57–65.

[26] N. TANOVIC-MILLER, *On strong summability*, Glas. Mat. Ser. III, **14** (1979), 87–97.

[27] Ş. YILDIZ, *On the absolute matrix summability factors of Fourier series*, Math. Notes **103** (2018), 297–303.

[28] Ş. YILDIZ, *On the generalizations of some factors theorems for infinite series and Fourier series*, Filomat **33** (2019), 4343–4351.

[29] Ş. YILDIZ, *Matrix application of power increasing sequences to infinite series and Fourier series*, Ukr. Math. J. **72** (2020), 730–740.

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